(a) We'll assume that the new temperature scale is related to the Celsius scale by a linear transformation; then \( T_S = mT_C + b \), where \( m \) and \( b \) are constants to be determined, \( T_S \) is the temperature measurement in the "new" scale, and \( T_C \) is the temperature measurement in Celsius degrees.

One of our known points is absolute zero;

\[
T_S = mT_C + b, \\
(0) = m(-273.15^\circ C) + b.
\]

We have two other points, the melting and boiling points for water,

\[
(T_S)_{bp} = m(100^\circ C) + b, \\
(T_S)_{mp} = m(0^\circ C) + b;
\]

we can subtract the top equation from the bottom equation to get

\[
(T_S)_{bp} - (T_S)_{mp} = 100 \, ^\circ C \cdot m.
\]

We are told this is 180 \( ^\circ S \), so \( m = 1.8 \, S^\circ /C^\circ \). Put this into the first equation and then find \( b \),

\[
b = 273.15^\circ C \cdot m = 491.67^\circ S.\]

The conversion is then

\[
T_S = (1.8 \, S^\circ /C^\circ)T_C + (491.67^\circ S).
\]

(b) The melting point for water is 491.67^\circ S; the boiling point for water is 180 \( ^\circ S \) above this, or 671.67^\circ S.

---

E21-2 \( T_F = 9(-273.15 \, \text{deg} C)/5 + 32^\circ F = -459.67^\circ F. \)

---

E21-3 (a) We'll assume that the new temperature scale is related to the Celsius scale by a linear transformation; then \( T_S = mT_C + b \), where \( m \) and \( b \) are constants to be determined, \( T_S \) is the temperature measurement in the "new" scale, and \( T_C \) is the temperature measurement in Celsius degrees.

One of our known points is absolute zero;

\[
T_S = mT_C + b, \\
(0) = m(-273.15^\circ C) + b.
\]

We have two other points, the melting and boiling points for water,

\[
(T_S)_{bp} = m(100^\circ C) + b, \\
(T_S)_{mp} = m(0^\circ C) + b;
\]

we can subtract the top equation from the bottom equation to get

\[
(T_S)_{bp} - (T_S)_{mp} = 100 \, ^\circ C \cdot m.
\]

We are told this is 100 \( ^\circ Q \), so \( m = 1.0 \, Q^\circ /C^\circ \). Put this into the first equation and then find \( b \),

\[
b = 273.15^\circ C = 273.15^\circ Q.\]

The conversion is then

\[
T_S = T_C + (273.15^\circ S).
\]

(b) The melting point for water is 273.15^\circ Q; the boiling point for water is 100 \( ^\circ Q \) above this, or 373.15^\circ Q.

(c) Kelvin Scale.
E21-4  (a) \( T = (9/5)(6000 K - 273.15) + 32 = 10000^\circ F \).  
(b) \( T = (5/9)(98.6^\circ F - 32) = 37.0^\circ C \).  
(c) \( T = (5/9)(-70^\circ F - 32) = -57^\circ C \).  
(d) \( T = (9/5)(-183^\circ C) + 32 = -297^\circ F \).  
(e) It depends on what you think is hot. My mom thinks 79\(^\circ F\) is too warm; that's \( T = (5/9)(79^\circ F - 32) = 26^\circ C \).  

E21-5  \( T = (9/5)(310 K - 273.15) + 32 = 98.3^\circ F \), which is fine.  

E21-6  (a) \( T = 2(5/9)(T - 32) \), so \(-T/10 = -32\), or \( T = 320^\circ F \).  
(b) \( 2T = (5/9)(T - 32) \), so \( 13T/5 = -32 \), or \( T = -12.3^\circ F \).  

E21-7  If the temperature (in Kelvin) is directly proportional to the resistance then \( T = kR \), where \( k \) is a constant of proportionality. We are given one point, \( T = 273.16 \) K when \( R = 90.35 \Omega \), but that is okay; we only have one unknown, \( k \). Then \( (273.16 \text{ K}) = k(90.35 \Omega) \) or \( k = 3.023 \text{ K}/\Omega \).  

If the resistance is measured to be \( R = 96.28 \Omega \), we have a temperature of  
\[ T = kR = (3.023 \text{ K}/\Omega)(96.28 \Omega) = 291.1 \text{ K}. \]

E21-8  \( T = (510^\circ C)/(0.028 V) V, so T = (1.82 \times 10^4 \text{C}/V)(0.0102 V) = 186^\circ C \).  

E21-9  We must first find the equation which relates gain to temperature, and then find the gain at the specified temperature. If we let \( G \) be the gain we can write this linear relationship as  
\[ G = mT + b, \]
where \( m \) and \( b \) are constants to be determined. We have two known points:  
\[ (30.0) = m(20.0^\circ C) + b, \]
\[ (35.2) = m(55.0^\circ C) + b. \]

If we subtract the top equation from the bottom we get \( 5.2 = m(35.0^\circ C) \), or \( m = 1.49 \text{C}^{-1} \). Put this into either of the first two equations and  
\[ (30.0) = (0.149 \text{C}^{-1})(20.0^\circ C) + b, \]
which has a solution \( b = 27.0 \).  

Now to find the gain when \( T = 28.0^\circ C \):  
\[ G = mT + b = (0.149 \text{C}^{-1})(28.0^\circ C) + (27.0) = 31.2 \]

E21-10  \( p/p_\text{tr} = (373.15 \text{K})/(273.16 \text{K}) = 1.366 \).  

E21-11  100 cm Hg is 1000 torr. \( P_\text{Hg} = (100 \text{ cm Hg})(373 \text{K})/(273.16 \text{K}) = 136.550 \text{ cm Hg} \). Nitrogen records a temperature which is 0.2 K higher, so \( P_\text{N} = (100 \text{ cm Hg})(373.2 \text{K})/(273.16 \text{K}) = 136.623 \text{ cm Hg} \). The difference is 0.073 cm Hg.  

E21-12  \( \Delta L = (23 \times 10^{-6}/\text{C}^\circ)(33 \text{ m})(15\text{C}^\circ) = 1.1 \times 10^{-2} \text{m} \).  

E21-13  \( \Delta L = (3.2 \times 10^{-6}/\text{C}^\circ)(200 \text{ in})(60\text{C}^\circ) = 3.8 \times 10^{-2} \text{in} \).
Consequently, the height of the liquid changes according to

\[
\Delta V = (h_0 + \Delta h)(A_0 + \Delta A) - h_0 A, \\
\approx h_0 \Delta A + A_0 \Delta h, \\
\Delta V/V_0 = \Delta A/A_0 + \Delta h/h_0.
\]

Then

\[
\Delta h = (1.28 \text{ m}/2)[(1.1 \times 10^{-5}/\text{C}^o) - (4.2 \times 10^{-5}/\text{C}^o)](13 \text{ C}^o) = 2.6 \times 10^{-4}\text{ m}.
\]

E21-36  (a) \( \beta = (dV/dT)/V \). If \( pV = nRT \), then \( p \, dV = nR \, dT \), so

\[
\beta = (nR/p)/V = nR/pV = 1/T.
\]

(b) Kelvins.

(c) \( \beta = 1/(300/\text{K}) = 3.3 \times 10^{-3}/\text{K} \).

E21-37  (a) \( V = (1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(273\text{K})/(1.01 \times 10^5\text{Pa}) = 2.25 \times 10^{-2}\text{ m}^3 \).

(b) \( (6.02 \times 10^{23}\text{mol}^{-1})/(2.25 \times 10^4/\text{cm}^3) = 2.68 \times 10^{19} \).

E21-38  \( n/V = p/kT \), so

\[
n/V = (1.01 \times 10^{-13}\text{Pa})/(1.38 \times 10^{-23}\text{J}/\text{K})(295\text{ K}) = 25 \text{ part/cm}^3.
\]

E21-39  (a) Using Eq. .21-17,

\[
n = \frac{pV}{RT} = \frac{(108 \times 10^3\text{Pa})(2.47 \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(12 + 273)\text{ K}} = 113 \text{ mol}.
\]

(b) Use the same expression again,

\[
V = \frac{nRT}{p} = \frac{(113 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(31 + 273)\text{ K}}{(316 \times 10^3\text{Pa})} = 0.903 \text{ m}^3.
\]

E21-40  (a) \( n = pV/RT = (1.01 \times 10^5\text{Pa})(1.13 \times 10^{-3}\text{m}^3)/(8.31 \text{ J/mol} \cdot \text{K})(315\text{K}) = 4.36 \times 10^{-2}\text{mol} \).

(b) \( T_f = T_i p_i V_i/p V_i, \) so

\[
T_f = \frac{(315\text{ K})(1.06 \times 10^5\text{Pa})(1.530 \times 10^{-3}\text{m}^3)}{(1.01 \times 10^5\text{Pa})(1.130 \times 10^{-3}\text{m}^3)} = 448\text{ K}.
\]

E21-41  \( p_i = (14.7 + 24.2) \text{ lb/in}^2 = 38.9 \text{ lb/in}^2 \). \( p_r = p_i T_i V_i/T_1 V_f, \) so

\[
p_r = \frac{(38.9 \text{ lb/in}^2)(299\text{K})(988 \text{ in}^3)}{(270\text{K})(1020 \text{ in}^3)} = 41.7 \text{ lb/in}^2.
\]

The gauge pressure is then \( (41.7 - 14.7) \text{ lb/in}^2 = 27.0 \text{ lb/in}^2 \).

E21-42  Since \( p = F/A \) and \( F = mg \), a reasonable estimate for the mass of the atmosphere is

\[
m = pA/g = (1.01 \times 10^5\text{Pa})4\pi(6.37 \times 10^8\text{m})^2/(9.81 \text{ m/s}^2) = 5.25 \times 10^{18}\text{kg}.
\]
E21-43 \( p = p_0 + \rho gh \), where \( h \) is the depth. Then \( P_f = 1.01 \times 10^5 \text{Pa} \) and
\[
p_i = (1.01 \times 10^5 \text{Pa}) + (998 \text{kg/m}^3)(9.81 \text{m/s}^2)(41.5 \text{m}) = 5.07 \times 10^5 \text{Pa}.
\]
\[V_f = V_i p_f T_f / p_i T_i, \text{ so}
V_f = \frac{(19.4 \text{ cm}^3)(5.07 \times 10^5 \text{Pa})}{(1.01 \times 10^5 \text{Pa})(277 \text{K})} = 104 \text{ cm}^3.
\]

E21-44 The new pressure in the pipe is
\[p_f = p_i V_i / V_f = (1.01 \times 10^5 \text{Pa})(2) = 2.02 \times 10^5 \text{Pa}.
\]
The water pressure at some depth \( y \) is given by \( p = p_0 + \rho gy \), so
\[y = \frac{(2.02 \times 10^5 \text{Pa}) - (1.01 \times 10^5 \text{Pa})}{(998 \text{kg/m}^3)(9.81 \text{m/s}^2)} = 10.3 \text{ m}.
\]
Then the water/air interface inside the tube is at a depth of 10.3 m; so \( h = (10.3 \text{ m}) + (25.0 \text{ m}) / 2 = 22.8 \text{ m} \).

P21-1 (a) The dimensions of \( A \) must be \([\text{time}]^{-1}, \) as can be seen with a quick inspection of the equation. We would expect that \( A \) would depend on the surface area at the very least; however, that means that it must also depend on some other factor to fix the dimensionality of \( A \).
(b) Rearrange and integrate,
\[
\int_{\Delta T_0}^T \frac{d\Delta T}{\Delta T} = -\int_0^t A \, dt;
\]
\[
\ln(\Delta T / \Delta T_0) = -At,
\]
\[
\Delta T = \Delta T_0 e^{-At}.
\]

P21-2 First find \( A \).
\[
A = \frac{\ln(\Delta T_0 / \Delta T)}{t} = \frac{\ln[(29 \text{ C}°)/(25 \text{ C}°)]}{(45 \text{ min})} = 3.30 \times 10^{-3} / \text{min}.
\]
Then find time to new temperature difference.
\[
t = \frac{\ln(\Delta T_0 / \Delta T)}{\ln[(29 \text{ C}°)/(21 \text{ C}°)]}{(3.30 \times 10^{-3} / \text{min})} = 97.8 \text{min}
\]
This happens 97.8 – 45 = 53 minutes later.

P21-3 If we neglect the expansion of the tube then we can assume the cross sectional area of the tube is constant. Since \( V = Ah \), we can assume that \( \Delta V = A \Delta h \). Then since \( \Delta V = \beta V_0 \Delta T \), we can write \( \Delta h = \beta h_0 \Delta T \).

P21-4 For either container we can write \( p_i V_i = n_i RT_i \). We are told that \( V_i \) and \( n_i \) are constants. Then \( \Delta p = AT_1 - BT_2 \), where \( A \) and \( B \) are constants. When \( T_1 = T_2 \), \( \Delta p = 0 \), so \( A = B \). When \( T_1 = T_{1r} \) and \( T_2 = T_b \) we have
\[
(120 \text{ mm Hg}) = A(373 \text{ K} - 273.16 \text{ K}),
\]
so \( A = 1.202 \text{ mm Hg/K} \). Then
\[
T = \frac{(90 \text{ mm Hg}) + (1.202 \text{ mm Hg/K})(273.16 \text{ K})}{(1.202 \text{ mm Hg/K})} = 348 \text{ K}.
\]
Actually, we could have assumed \( A \) was negative, and then the answer would be 198 K.

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Start with a differential form for Eq. 21-8, \( dL/dT = \alpha L_0 \), rearrange, and integrate:

\[
\int_{L_0}^{L} dL = \int_{T_0}^{T} \alpha L_0 \, dT,
\]

\[
L - L_0 = L_0 \int_{T_0}^{T} \alpha \, dT,
\]

\[
L = L_0 \left( 1 + \int_{T_0}^{T} \alpha \, dT \right).
\]

\[\Delta L = \alpha L_0 \Delta T,\]

so

\[
\Delta T = \frac{1}{\alpha L} \frac{\Delta L}{\Delta t} = \left( \frac{96 \times 10^{-9} \text{m/s}}{(23 \times 10^{-6}/\text{C}^\circ)(1.8 \times 10^{-2} \text{m})} \right) = 0.23^\circ\text{C/s}.
\]

(a) Consider the work that was done for Ex. 21-27. The length of rod \( a \) is

\[L_a = L_{a,0}(1 + \alpha_a \Delta T),\]

while the length of rod \( b \) is

\[L_b = L_{b,0}(1 + \alpha_b \Delta T).
\]

The difference is

\[L_a - L_b = L_{a,0}(1 + \alpha_a \Delta T) - L_{b,0}(1 + \alpha_b \Delta T),\]

\[= L_{a,0} - L_{b,0} + (L_{a,0} \alpha_a - L_{b,0} \alpha_b) \Delta T,\]

which will be a constant is

\[L_{a,0} \alpha_a = L_{b,0} \alpha_b \text{ or } L_{a,0} \alpha_a \propto 1/\alpha_i.
\]

(b) We want \( L_{a,0} - L_{b,0} = 0.30 \text{ m} \) so

\[k/\alpha_a - k/\alpha_b = 0.30 \text{ m},\]

where \( k \) is a constant of proportionality;

\[k = (0.30 \text{ m}) / (1/(11 \times 10^{-6} / \text{C}^\circ) - 1/(19 \times 10^{-6} / \text{C}^\circ)) = 7.84 \times 10^{-6} \text{ m/C}^\circ.
\]

The two lengths are

\[L_a = (7.84 \times 10^{-6} \text{ m/C}^\circ)/(11 \times 10^{-6} / \text{C}^\circ) = 0.713 \text{ m}\]

for steel and

\[L_b = (7.84 \times 10^{-6} \text{ m/C}^\circ)/(19 \times 10^{-6} / \text{C}^\circ) = 0.413 \text{ m}\]

for brass.

The fractional increase in length of the bar is \( \Delta L/L_0 = \alpha \Delta T \). The right triangle on the left has base \( L_0/2 \), height \( x \), and hypotenuse \((L_0 + \Delta L)/2\). Then

\[x = \frac{1}{2} \sqrt{(L_0 + \Delta L)^2 - L_0^2} = \frac{L_0}{2} \sqrt{2 \frac{\Delta L}{L_0}}.
\]

With numbers,

\[x = \frac{(3.77 \text{ m})}{2} \sqrt{2(25 \times 10^{-6} / \text{C}^\circ)(32 \text{ C}^\circ)} = 7.54 \times 10^{-2} \text{ m}.
\]
P21-9 We want to evaluate $V = V_0(1 + \int \beta dT)$; the integral is the area under the graph; the graph looks like a triangle, so the result is

$$V = V_0[1 + (16 \, C^\circ)(0.0002/C^\circ)/2] = (1.0016)V_0.$$  

The density is then

$$\rho = \rho_0(V_0/V) = (1000 \, kg/m^3)/(1.0016) = 0.9984 \, kg/m^3.$$  

P21-10 At 0.00°C the glass bulb is filled with mercury to a volume $V_0$. After heating, the difference in volume changes is given by

$$\Delta V = V_0(\beta - 3\alpha)\Delta T.$$  

Since $T_0 = 0.0^\circ C$, then $\Delta T = T$, if it is measured in °C. The amount of mercury in the capillary is $\Delta V$, and since the cross sectional area is fixed at $A$, then the length is $L = \Delta V/A$, or

$$L = \frac{V}{A}(\beta - 3\alpha)\Delta T.$$  

P21-11 Let $a$, $b$, and $c$ correspond to aluminum, steel, and invar, respectively. Then

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$  

We can replace $a$ with $a_0(1 + \alpha_a \Delta T)$, and write similar expressions for $b$ and $c$. Since $a_0 = b_0 = c_0$, this can be simplified to

$$\cos C \approx \frac{1}{2} + \frac{1}{2}(\alpha_a + \alpha_b - 2\alpha_c)\Delta T.$$  

Expand this as a Taylor series in terms of $\Delta T$, and we find

$$\cos C \approx \frac{1}{2} + \frac{1}{2}(\alpha_a + \alpha_b - 2\alpha_c)\Delta T.$$  

Now solve:

$$\Delta T = \frac{2\cos(59.95^\circ) - 1}{(23\times10^{-6}/C^\circ) + (11\times10^{-6}/C^\circ) - 2(0.7\times10^{-6}/C^\circ)} = 46.4^\circ C.$$  

The final temperature is then 66.4°C.

P21-12 The bottom of the iron bar moves downward according to $\Delta L = \alpha L \Delta T$. The center of mass of the iron bar is located in the center; it moves downward half the distance. The mercury expands in the glass upwards; subtracting off the distance the iron moves we get

$$\Delta h = \beta h \Delta T - \Delta L = (\beta h - \alpha L)\Delta T.$$  

The center of mass in the mercury is located in the center. If the center of mass of the system is to remain constant we require

$$m_i \Delta L/2 = m_m(\Delta h - \Delta L)/2;$$  

or, since $\rho = mV = mAy$,

$$\rho_i \alpha L = \rho_m(\beta h - 2\alpha L).$$  

Solving for $h$,

$$h = \frac{(12\times10^{-6}/C^\circ)(1.00 \, m)[(7.87\times10^3 \, kg/m^3) + 2(13.6\times10^3 \, kg/m^3)]}{(13.6\times10^3 \, kg/m^3)(18\times10^{-6}/C^\circ)} = 0.17 \, m.$$  

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E22-1  (a) \( n = (2.56 \text{ g})/(197 \text{ g/mol}) = 1.30 \times 10^{-2} \text{ mol.} \)
(b) \( N = (6.02 \times 10^{23} \text{ mol}^{-1})(1.30 \times 10^{-2} \text{ mol}) = 7.83 \times 10^{21} \).

E22-2  (a) \( N = pV/kT = (1.01 \times 10^{5} \text{ Pa})(1.00 \text{ m}^3)/(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K}) = 2.50 \times 10^{25} \).
(b) \( n = (2.50 \times 10^{25})/(6.02 \times 10^{23} \text{ mol}^{-1}) = 41.5 \text{ mol.} \) Then
\[ m = (41.5 \text{ mol})[75\% (28 \text{ g/mol}) + 25\% (32 \text{ g/mol})] = 1.20 \text{ kg}. \]

E22-3  (a) We first need to calculate the molar mass of ammonia. This is
\( M = M(\text{N}) + 3M(\text{H}) = (14.0 \text{ g/mol}) + 3(1.01 \text{ g/mol}) = 17.0 \text{ g/mol} \).
The number of moles of nitrogen present is
\[ n = m/M = (315 \text{ g})/(17.0 \text{ g/mol}) = 18.5 \text{ mol.} \]

The volume of the tank is
\[ V = nRT/p = (18.5 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(350 \text{ K})/(1.35 \times 10^{6} \text{ Pa}) = 3.99 \times 10^{-2} \text{ m}^3. \]
(b) After the tank is checked the number of moles of gas in the tank is
\[ n = pV/(RT) = (8.68 \times 10^{5} \text{ Pa})(3.99 \times 10^{-2} \text{ m}^3)/[(8.31 \text{ J/mol} \cdot \text{K})(295 \text{ K})] = 14.1 \text{ mol}. \]
In that case, 4.4 mol must have escaped; that corresponds to a mass of
\[ m = nM = (4.4 \text{ mol})(17.0 \text{ g/mol}) = 74.8 \text{ g}. \]

E22-4  (a) The volume per particle is \( V/N = kT/P \), so
\[ V/N = (1.38 \times 10^{-23} \text{ J/K})(285 \text{ K})/(1.01 \times 10^{5} \text{ Pa}) = 3.89 \times 10^{-26} \text{ m}^3. \]
The edge length is the cube root of this, or \( 3.39 \times 10^{-9} \text{ m}. \) The ratio is 11.3.
(b) The volume per particle is \( V/N_A \), so
\[ V/N_A = (18 \times 10^{-6} \text{ m}^3)/(6.02 \times 10^{23}) = 2.99 \times 10^{-29} \text{ m}^3. \]
The edge length is the cube root of this, or \( 3.10 \times 10^{-10} \text{ m}. \) The ratio is 1.03.

E22-5  The volume per particle is \( V/N = kT/P \), so
\[ V/N = (1.38 \times 10^{-23} \text{ J/K})(308 \text{ K})/(1.22)(1.01 \times 10^{5} \text{ Pa}) = 3.45 \times 10^{-26} \text{ m}^3. \]
The fraction actually occupied by the particle is
\[ \frac{4\pi(0.710 \times 10^{-10} \text{ m}^3)/3}{(3.45 \times 10^{-26} \text{ m}^3)} = 4.34 \times 10^{-5}. \]

E22-6  The component of the momentum normal to the wall is
\[ p_y = (3.3 \times 10^{-27} \text{ kg})(1.0 \times 10^5 \text{ m/s}) \cos(55^\circ) = 1.89 \times 10^{-24} \text{ kg} \cdot \text{m/s}. \]
The pressure exerted on the wall is
\[ p = \frac{F}{A} = \frac{(1.6 \times 10^{22} \text{ s})2(1.89 \times 10^{-24} \text{ kg} \cdot \text{m/s})}{(2.0 \times 10^{-4} \text{ m}^2)} = 3.0 \times 10^3 \text{ Pa}. \]
(a) From Eq. 22-9,

\[ v_{\text{rms}} = \sqrt{\frac{3p}{\rho}}. \]

Then

\[ p = 1.23 \times 10^{-3} \text{ atm} \left(1 \times 10^6 \text{ Pa} \right) = 124 \text{ Pa} \]

and

\[ \rho = 1.32 \times 10^{-5} \text{ g/cm}^3 \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^3 = 1.32 \times 10^{-2} \text{ kg/m}^3. \]

Finally,

\[ v_{\text{rms}} = \sqrt{\frac{3(1240 \text{ Pa})}{(1.32 \times 10^{-2} \text{ kg/m}^3)}} = 531 \text{ m/s}. \]

(b) The molar density of the gas is just \( n/V \), but this can be found quickly from the ideal gas law as

\[ \frac{n}{V} = \frac{p}{RT} = \frac{1240 \text{ Pa}}{(8.31 \text{ J/mol} \cdot \text{K})(317 \text{ K})} = 4.71 \times 10^{-1} \text{ mol/m}^3. \]

(c) We were given the density, which is mass per volume, so we could find the molar mass from

\[ \frac{n}{V} = \frac{(1.32 \times 10^{-2} \text{ kg/m}^3)}{(4.71 \times 10^{-1} \text{ mol/m}^3)} = 28.0 \text{ g/mol}. \]

But what gas is it? It could contain any atom lighter than silicon; trial and error is the way to go. Some of my guesses include \( \text{C}_2\text{H}_4 \) (ethene), \( \text{CO} \) (carbon monoxide), and \( \text{N}_2 \). There’s no way to tell which is correct at this point; in fact, the gas could be a mixture of all three.

E22-8 The density is \( \rho = n/V = nM_r/V \), or

\[ \rho = (0.350 \text{ mol})(0.0280 \text{ kg/mol})/(\pi(0.125 \text{ m}/2)^2(0.560 \text{ m}) = 1.43 \text{ kg/m}^3. \]

The rms speed is

\[ v_{\text{rms}} = \sqrt{\frac{3(2.05)(1.01 \times 10^6 \text{ Pa})}{(1.43 \text{ kg/m}^3)}} = 659 \text{ m/s}. \]

E22-9 (a) \( N/V = p/kT = (1.01 \times 10^5 \text{ Pa})/(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K}) = 2.68 \times 10^{25} \text{ /m}^3. \)

(b) Note that Eq. 22-11 is wrong; for the explanation read the last two paragraphs in the first column on page 502. We need an extra factor of \( \sqrt{2} \), so \( \pi d^2 = V/\sqrt{2}N\lambda \), so

\[ d = \sqrt{1/\sqrt{2}\pi(2.68 \times 10^{25} \text{ /m}^3)(285 \times 10^{-9} \text{ m})} = 1.72 \times 10^{-10} \text{ m}. \]

E22-10 (a) \( \lambda = V/\sqrt{2}N\pi d^2 \), so

\[ \lambda = \frac{1}{\sqrt{2}(1.0 \times 10^6 \text{ /m}^3)\pi(2.0 \times 10^{-10} \text{ m})^2} = 5.6 \times 10^{12} \text{ m}. \]

(b) Particles effectively follow ballistic trajectories.
E22-20  We want to integrate

\[(v^2)_{av} = \frac{1}{N} \int_0^\infty N(v)v^2 \, dv;\]

\[= \frac{1}{N} \int_0^\infty 4\pi N \left( \frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT} \, dv;\]

\[= 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} \int_0^\infty v^2 e^{-mv^2/2kT} \, dv.\]

The easiest way to attack this is first with a change of variables— let \(x^2 = mv^2/2kT\), then \(\sqrt{2kT/m} \, dx = dv\). The limits of integration don’t change. Then

\[(v^2)_{av} = 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} \int_0^\infty \left( \frac{2kT}{m} \right)^{5/2} x^4 e^{-x^2} \, dx;\]

\[= \frac{8kT}{\sqrt{\pi m}} \int_0^\infty x^6 e^{-x^2} \, dx.\]

Look up the integral; although you can solve it by first applying a Feynman trick (see solution to Exercise 22-21) and then squaring the integral and changing to polar coordinates. I looked it up. I found \(3\sqrt{\pi}/8\), so

\[(v^2)_{av} = \frac{8kT}{\sqrt{\pi m}} \cdot \frac{3\sqrt{\pi}}{8} = 3kT/m.\]

E22-21  Apply Eq. 22-20:

\[v_{rms} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{J/K})(287 \text{K})/(5.2 \times 10^{-17} \text{kg})}{(5.2 \times 10^{-17} \text{kg})}} = 1.5 \times 10^{-2} \text{m/s}.\]

E22-22  Since \(v_{rms} \propto \sqrt{T/m}\), we have

\[T = (299 \text{K})(4/2) = 598 \text{K},\]

or 325°C.

E22-23  (a) The escape speed is found on page 310; \(v = 11.2 \times 10^3 \text{m/s}\). For hydrogen,

\[T = (2)(1.67 \times 10^{-27} \text{kg})(11.2 \times 10^3 \text{m/s})^2 / 3(1.38 \times 10^{-23} \text{J/K}) = 1.0 \times 10^4 \text{K}.\]

For oxygen,

\[T = (32)(1.67 \times 10^{-27} \text{kg})(11.2 \times 10^3 \text{m/s})^2 / 3(1.38 \times 10^{-23} \text{J/K}) = 1.6 \times 10^5 \text{K}.\]

(b) The escape speed is found on page 310; \(v = 2.38 \times 10^3 \text{m/s}\). For hydrogen,

\[T = (2)(1.67 \times 10^{-27} \text{kg})(2.38 \times 10^3 \text{m/s})^2 / 3(1.38 \times 10^{-23} \text{J/K}) = 460 \text{K}.\]

For oxygen,

\[T = (32)(1.67 \times 10^{-27} \text{kg})(2.38 \times 10^3 \text{m/s})^2 / 3(1.38 \times 10^{-23} \text{J/K}) = 7300 \text{K}.\]

(c) There should be more oxygen than hydrogen.

E22-24  (a) \(v_{av} = (70 \text{ km/s})/(22) = 3.18 \text{ km/s}.

(b) \(v_{rms} = \sqrt{(250 \text{ km}^2/\text{s}^2)/(22)} = 3.37 \text{ km/s}.

(c) 3.0 \text{ km/s}.
According to the equation directly beneath Fig. 22-8,
\[ \omega = \nu \phi/L = (212 \text{ m/s})(0.0841 \text{ rad})/(0.204 \text{ m}) = 87.3 \text{ rad/s}. \]

If \( v_p = v_{rms} \) then \( 2T_2 = 3T_1 \), or \( T_2/T_1 = 3/2 \).

Read the last paragraph on the first column of page 505. The distribution of speeds is proportional to
\[ \nu^3 e^{-\frac{mv^2}{2kT}} = \nu^3 e^{-Bv^2}, \]
taking the derivative \( dN/dv \) and setting equal to zero yields
\[ \frac{dN}{dv} = 3\nu^2 e^{-Bv^2} - 2B\nu^4 e^{-Bv^2}, \]
and setting this equal to zero,
\[ \nu^2 = \frac{3}{2B} = 3kT/m. \]

(a) \( v = \sqrt{3(8.31 J/mol \cdot K)(4220 K)/(0.07261 \text{ kg/mol})} = 1200 \text{ m/s}. \)
(b) Half of the sum of the diameters, or 273 pm.
(c) The mean free path of the germanium in the argon is
\[ \lambda = 1/\sqrt[3]{(4.13 \times 10^{25} / \text{m}^3)\pi(273 \times 10^{-12} \text{m})^2} = 7.31 \times 10^{-8} \text{m}. \]
The collision rate is
\[ (1200 \text{ m/s})/(7.31 \times 10^{-8} \text{m}) = 1.64 \times 10^{10} / \text{s}. \]

The fraction of particles that interests us is
\[ \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \int_{0.01kT}^{0.03kT} E^{1/2} e^{-E/kT} dE. \]
Change variables according to \( E/kT = x \), so that \( dE = kT dx \). The integral is then
\[ \frac{2}{\sqrt{\pi}} \int_{0.01}^{0.03} x^{1/2} e^{-x} dx. \]
Since the value of \( x \) is so small compared to 1 throughout the range of integration, we can expand according to
\[ e^{-x} \approx 1 - x \text{ for } x \ll 1. \]
The integral then simplifies to
\[ \frac{2}{\sqrt{\pi}} \int_{0.01}^{0.03} x^{1/2}(1 - x) dx = \frac{2}{\sqrt{\pi}} \left[ \frac{2}{3} x^{3/2} - \frac{2}{5} x^{5/2} \right]_{0.01}^{0.03} = 3.09 \times 10^{-3}. \]

Write \( N(E) = N(E_p + \epsilon) \). Then
\[ N(E_p + \epsilon) \approx N(E_p) + \epsilon \frac{dN(E)}{dE} \Bigg|_{E_p} + \ldots \]
But the very definition of \( E_p \) implies that the first derivative is zero. Then the fraction of particles with energies in the range \( E_p \pm 0.02kT \) is
\[ \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} (kT/2)^{1/2} e^{-1/2}(0.02kT), \]
or \( 0.04 \sqrt{1/2 \epsilon \pi} = 9.68 \times 10^{-3} \).
As expected, the average speed is less than the maximum speed. We can make a prediction about the root mean square speed; it will be larger than the average speed (see Exercise 22-15 above) but smaller than the maximum speed.

(c) The root-mean-square velocity is found from

\[ v_{\text{rms}}^2 = \frac{1}{N} \int_0^\infty N(v) v^2 \, dv. \]

Using our results from above,

\[ v_{\text{rms}}^2 = \frac{1}{N} \int_0^{v_0} \left( \frac{3N}{v_0^3} v^2 \right) v^2 \, dv \]
\[ = \frac{3}{v_0^3} \int_0^{v_0} v^4 \, dv = \frac{3 v_0^5}{5} = \frac{3}{5} v_0^2. \]

Then, taking the square root,

\[ v_{\text{rms}} = \sqrt{\frac{3}{5}} v_0. \]

Is \( \sqrt{3/5} > 3/4 \)? It had better be.

P22-10
P22-11
P22-12
P22-13
P22-14

**P22-15** The mass of air displaced by 2180 m\(^3\) is \( m = (1.22 \, \text{kg/m}^3)(2180 \, \text{m}^3) = 2660 \, \text{kg} \). The mass of the balloon and basket is 249 kg and we want to lift 272 kg; this leaves a remainder of 2140 kg for the mass of the air inside the balloon. This corresponds to \((2140 \, \text{kg})/(0.0289 \, \text{kg/mol}) = 7.4 \times 10^4 \, \text{mol} \).

The temperature of the gas inside the balloon is then

\[ T = \frac{pV}{nR} = \frac{[(1.01 \times 10^5 \, \text{Pa})(2180 \, \text{m}^3)]}{[(7.4 \times 10^4 \, \text{mol})(8.31 \, \text{J/mol} \cdot \text{K})]} = 358 \, \text{K}. \]

That's 85°C.

P22-16
P22-17
E23-1 We apply Eq. 23-1,
\[ H = kA \frac{\Delta T}{\Delta x} \]
The rate at which heat flows out is given as a power per area (mW/m²), so the quantity given is really \( H/A \). Then the temperature difference is
\[ \Delta T = \frac{H}{A} \frac{\Delta x}{k} = 0.054 \text{ W/m}^2 (33,000 \text{ m}) \frac{2.5 \text{ W/m} \cdot \text{K}}{(2.5 \text{ W/m} \cdot \text{K})} = 710 \text{ K} \]
The heat flow is out, so that the temperature is higher at the base of the crust. The temperature there is then
\[ 710 + 10 = 720 \, ^\circ \text{C}. \]

E23-2 We apply Eq. 23-1,
\[ H = kA \Delta T = (0.74 \text{ W/m} \cdot \text{K})(6.2 \text{ m})(3.8 \text{ m}) \frac{44 \text{ C}^\circ}{0.32 \text{ m}} = 2400 \text{ W.} \]

E23-3 (a) \( \Delta T/\Delta x = (136 \text{ C}^\circ)/(0.249 \text{ m}) = 546 \text{ C}^\circ/\text{m}. \)
(b) \( H = kA\Delta T/\Delta x = (401 \text{ W/m} \cdot \text{K})(1.80 \text{ m}^2)/(546 \text{ C}^\circ/\text{m}) = 3.94 \times 10^5 \text{ W.} \)
(c) \( T_H = (-12\text{ C}^\circ + 136 \text{ C}^\circ) = 124\text{ C}^\circ. \) Then
\[ T = (124\text{ C}^\circ) - (546 \text{ C}^\circ/\text{m})(0.11 \text{ m}) = 63.9\text{ C}^\circ. \]

E23-4 (a) \( H = (0.040 \text{ W/m} \cdot \text{K})(1.80 \text{ m}^2)(32. \text{ C}^\circ)/(0.012 \text{ m}) = 190 \text{ W.} \)
(b) Since \( k \) has increased by a factor of \((0.60)/(0.04) = 15\) then \( H \) should also increase by a factor of 15.

E23-5 There are three possible arrangements: a sheet of type 1 with a sheet of type 1; a sheet of type 2 with a sheet of type 2; and a sheet of type 1 with a sheet of type 2. We can look back on Sample Problem 23-1 to see how to start the problem; the heat flow will be
\[ H_{12} = \frac{A\Delta T}{(L/k_1) + (L/k_2)} \]
for substances of different types; and
\[ H_{11} = \frac{A\Delta T/L}{(L/k_1) + (L/k_1)} = \frac{A\Delta T k_1}{2L} \]
for a double layer of substance 1. There is a similar expression for a double layer of substance 2.

For configuration (a) we then have
\[ H_{11} + H_{22} = \frac{1}{2} \frac{A\Delta T k_1}{L} + \frac{1}{2} \frac{A\Delta T k_2}{L} = \frac{A\Delta T}{2L}(k_1 + k_2), \]
while for configuration (b) we have
\[ H_{12} + H_{21} = 2 \frac{A\Delta T}{(L/k_1) + (L/k_2)} = \frac{2A\Delta T}{L} ((1/k_1) + (1/k_2))^{-1}. \]

We want to compare these, so expanding the relevant part of the second configuration
\[ ((1/k_1) + (1/k_2))^{-1} = ((k_1 + k_2)/(k_1k_2))^{-1} = \frac{k_1k_2}{k_1 + k_2}. \]

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Then which is larger

\[(k_1 + k_2)/2 \text{ or } \frac{2k_1 k_2}{k_1 + k_2}\]

If \(k_1 \gg k_2\) then the expression becomes

\[k_1/2 \text{ and } 2k_2,\]

so the first expression is larger, and therefore configuration (b) has the lower heat flow. Notice that we get the same result if \(k_1 \ll k_2\)!

**E23-6** There's a typo in the exercise.

\[H = A \Delta T/R; \text{ since the heat flows through one slab and then through the other, we can write}\]

\[(T_1 - T_2)/R_1 = (T_2 - T_3)/R_2. \text{ Rearranging,}\]

\[T_1 = (T_1 R_2 + T_2 R_1)/(R_1 + R_2)\]

**E23-7** Use the results of Exercise 23-6. At the interface between ice and water \(T_2 = 0^\circ\text{C}\). Then \(R_1 T_2 + R_2 T_1 = 0\), or \(k_1 T_1/L_1 + k_2 T_2/L_2 = 0\). Not only that, \(L_1 + L_2 = L\), so

\[k_1 T_1 L_2 + (L - L_2)k_2 T_2 = 0\]

so

\[L_2 = \frac{(1.42 \text{ m})(1.67 \text{ W/m} \cdot \text{K})(-5.20^\circ\text{C})}{(1.67 \text{ W/m} \cdot \text{K})(-5.20^\circ\text{C}) - (0.502 \text{ W/m} \cdot \text{K})(3.98^\circ\text{C})} = 1.15 \text{ m.}\]

**E23-8** \(\Delta T\) is the same in both cases. So is \(k\). The top configuration has \(H_1 = kA \Delta T/(2L)\). The bottom configuration has \(H_2 = k(2A) \Delta T/L\). The ratio of \(H_b/H_t = 4\), so heat flows through the bottom configuration at 4 times the rate of the top. For the top configuration \(H_1 = (10 \text{ J})/(2 \text{ min}) = 5 \text{ J/min}\). Then \(H_b = 20 \text{ J/min}\). It will take

\[t = (30 \text{ J})/(20 \text{ J/min}) = 1.5 \text{ min.}\]

**E23-9** (a) This exercise has a distraction: it asks about the heat flow through the window, but what you need to find first is the heat flow through the air near the window. We are given the temperature gradient both inside and outside the window. Inside,

\[\frac{\Delta T}{\Delta x} = \frac{(20^\circ\text{C}) - (5^\circ\text{C})}{(0.08 \text{ m})} = 190 \text{ C}^\circ/\text{m};\]

a similar expression exists for outside.

From Eq. 23-1 we find the heat flow through the air;

\[H = kA \frac{\Delta T}{\Delta x} = (0.026 \text{ W/m} \cdot \text{K})(0.6 \text{ m})^2(190 \text{ C}^\circ/\text{m}) = 1.8 \text{ W}.\]

The value that we arrived at is the rate that heat flows through the air across an area the size of the window on either side of the window. This heat flow had to occur through the window as well, so

\[H = 1.8 \text{ W}\]

answers the window question.

(b) Now that we know the rate that heat flows through the window, we are in a position to find the temperature difference across the window. Rearranging Eq. 32-1,

\[\Delta T = \frac{H \Delta x}{kA} = \frac{(1.8 \text{ W})(0.005 \text{ m})}{(1.0 \text{ W/m} \cdot \text{K})(0.6 \text{ m})^2} = 0.025 \text{ C}^\circ,\]

so we were well justified in our approximation that the temperature drop across the glass is very small.

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E23-10  (a) \( W = +214 \ J \), done on means positive.
(b) \( Q = -293 \ J \), extracted from means negative.
(c) \( \Delta E_{\text{int}} = Q + W = (-293 \ J) + (+214 \ J) = -79.0 \ J \).

E23-11  (a) \( \Delta E_{\text{int}} \) along any path between these two points is
\[ \Delta E_{\text{int}} = Q + W = (50 \ J) + (-20 \ J) = 30 \ J. \]

Then along ibf \( W = (30 \ J) - (36 \ J) = -6 \ J \).
(b) \( Q = (-30 \ J) - (+13 \ J) = -43 \ J \).
(c) \( E_{\text{int},f} = E_{\text{int},i} + \Delta E_{\text{int}} = (10 \ J) + (30 \ J) = 40 \ J \).
(d) \( \Delta E_{\text{int}bf} = (22 \ J) - (10 \ J) = 12 \ J \); while \( \Delta E_{\text{int}ib} = (40 \ J) - (22 \ J) = 18 \ J \). There is no work done on the path bf, so
\[ Q_{bf} = \Delta E_{\text{int}bf} - W_{bf} = (18 \ J) - (0) = 18 \ J, \]
and \( Q_{ib} = Q_{ibf} - Q_{bf} = (36 \ J) - (18 \ J) = 18 \ J \).

E23-12  \( Q = mL = (0.10)(2.1 \times 10^6 \ kg)(333 \times 10^3 \ J/kg) = 7.0 \times 10^{12} \ J \).

E23-13  We don’t need to know the outside temperature because the amount of heat energy required is explicitly stated: 5.22 GJ. We just need to know how much water is required to transfer this amount of heat energy. Use Eq. 23-11, and then
\[ m = \frac{Q}{c\Delta T} = \frac{(5.22 \times 10^9 \ J)}{(4190 \ J/kg \cdot K)(50.0^\circ C - 22.0^\circ C)} = 4.45 \times 10^4 \ kg. \]
This is the mass of the water, we want to know the volume, so we’ll use the density, and then
\[ V = \frac{m}{\rho} = \frac{(4.45 \times 10^4 \ kg)}{(998 \ kg/m^3)} = 44.5 \ m^3. \]

E23-14  The heat energy required is \( Q = mc\Delta T \). The time required is \( t = Q/P \). Then
\[ t = \frac{(0.136 \ kg)(4190 \ J/kg \cdot K)(100^\circ C - 23.5^\circ C)}{(220 \ W)} = 198 \ s. \]

E23-15  \( Q = mL \), so \( m = (50.4 \times 10^3 \ J)/(333 \times 10^3 \ J/kg) = 0.151 \ kg \) is the amount which freezes.
Then \( (0.258 \ kg) - (0.151 \ kg) = 0.107 \ kg \) is the amount which does not freeze.

E23-16  (a) \( W = mg\Delta y; \) if \( |Q| = |W| \), then
\[ \Delta T = \frac{mg\Delta y}{mc} = \frac{(9.81 \ m/s^2)(49.4 \ m)}{(4190 \ J/kg \cdot K)} = 0.112^\circ C. \]

E23-17  There are three “things” in this problem: the copper bowl (b), the water (w), and the copper cylinder (c). The total internal energy changes must add up to zero, so
\[ \Delta E_{\text{int},b} + \Delta E_{\text{int},w} + \Delta E_{\text{int},c} = 0. \]
As in Sample Problem 23-3, no work is done on any object, so
\[ Q_b + Q_w + Q_c = 0. \]

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The heat transfers for these three objects are

\[ Q_b = m_b c_b (T_{f,b} - T_{i,b}), \]
\[ Q_w = m_w c_w (T_{f,w} - T_{i,w}) + L_v m_2, \]
\[ Q_c = m_c c_c (T_{f,c} - T_{i,c}). \]

For the most part, this looks exactly like the presentation in Sample Problem 23-3; but there is an extra term in the second line. This term reflects the extra heat required to vaporize \( m_2 = 4.70 \) g of water at 100°C into steam 100°C.

Some of the initial temperatures are specified in the exercise: \( T_{i,b} = T_{i,w} = 21.0 \)°C and \( T_{f,b} = T_{f,w} = T_{f,c} = 100 \)°C.

(a) The heat transferred to the water, then, is

\[ Q_w = (0.223 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) ((100 \text{°C}) - (21.0 \text{°C})), \]
\[ + (2.26 \times 10^6 \text{ J/kg})(4.70 \times 10^{-3} \text{ kg}), \]
\[ = 8.44 \times 10^4 \text{ J}. \]

This answer differs from the back of the book. I think that they (or was it me) used the latent heat of fusion when they should have used the latent heat of vaporization!

(b) The heat transferred to the bowl, then, is

\[ Q_w = (0.146 \text{ kg})(387 \text{ J/kg} \cdot \text{K}) ((100 \text{°C}) - (21.0 \text{°C})) = 4.46 \times 10^3 \text{ J}. \]

(c) The heat transferred from the cylinder was transferred into the water and bowl, so

\[ Q_c = -Q_b - Q_w = -(4.46 \times 10^3 \text{ J}) - (8.44 \times 10^4 \text{ J}) = -8.89 \times 10^4 \text{ J}. \]

The initial temperature of the cylinder is then given by

\[ T_{i,c} = T_{f,c} - \frac{Q_c}{m_c c_c} = (100 \text{°C}) - \frac{(-8.89 \times 10^4 \text{ J})}{(0.314 \text{ kg})(387 \text{ J/kg} \cdot \text{K})} = 832 \text{°C}. \]

E23-18 The temperature of the silver must be raised to the melting point and then the heated silver needs to be melted. The heat required is

\[ Q = mL + mc\Delta T = (0.130 \text{ kg})[(105 \times 10^3 \text{ J/kg}) + (236 \text{ J/kg} \cdot \text{K})(1235 \text{ K} - 289 \text{ K})] = 4.27 \times 10^4 \text{ J}. \]

E23-19 (a) Use \( Q = mc\Delta T \), \( m = \rho V \), and \( t = Q/P \). Then

\[ t = \frac{[m_a c_a + \rho_w V_w c_w] \Delta T}{P} \]
\[ = \frac{[(0.56 \text{ kg})(900 \text{ J/kg} \cdot \text{K}) + (998 \text{ kg/m}^3)(0.64 \times 10^{-3} \text{ m}^3)(4190 \text{ J/kg} \cdot \text{K})](100 \text{°C} - 12 \text{°C})}{(2400 \text{ W})} = 117 \text{ s}. \]

(b) Use \( Q = mL \), \( m = \rho V \), and \( t = Q/P \). Then

\[ t = \frac{\rho_w V_w L_w}{P} = \frac{(998 \text{ kg/m}^3)(0.640 \times 10^{-3} \text{ m}^3)(2256 \times 10^3 \text{ J/kg})}{(2400 \text{ W})} = 600 \text{ s} \]

is the additional time required.
The heat given off by the steam will be
\[ Q_s = m_s L_v + m_w c_w (50 \degree C). \]
The heat taken in by the ice will be
\[ Q_i = m_i L_f + m_i c_w (50 \degree C). \]
Equating,
\[ m_s = m_w \frac{L_f + c_w (50 \degree C)}{L_v + c_w (50 \degree C)} \]
\[ = (0.150 \text{ kg}) \frac{(333 \times 10^3 \text{ J/kg}) + (4190 \text{ J/kg-K})(50 \degree C)}{(2256 \times 10^3 \text{ J/kg}) + (4190 \text{ J/kg-K})(50 \degree C)} = 0.033 \text{ kg}. \]

The linear dimensions of the ring and sphere change with the temperature change according to
\[ \Delta d_r = \alpha_r d_r (T_{f,r} - T_{i,r}), \]
\[ \Delta d_s = \alpha_s d_s (T_{f,s} - T_{i,s}). \]

When the ring and sphere are at the same (final) temperature the ring and the sphere have the same diameter. This means that
\[ d_r + \Delta d_r = d_s + \Delta d_s \]
when \( T_{f,s} = T_{f,r} \). We'll solve these expansion equations first, and then go back to the heat equations.
\[ d_r + \Delta d_r = d_s + \Delta d_s, \]
\[ d_r (1 + \alpha_r (T_{f,r} - T_{i,r})) = d_s (1 + \alpha_s (T_{f,s} - T_{i,s})), \]
which can be rearranged to give
\[ \alpha_r d_r (T_{f,r} - T_{i,r}) = \alpha_s d_s (T_{f,s} - T_{i,s}). \]
but since the final temperatures are the same,
\[ T_f = \frac{d_s (1 - \alpha_s T_{i,s}) - d_r (1 - \alpha_r T_{i,r})}{\alpha_r d_r - \alpha_s d_s}. \]
Putting in the numbers,
\[ T_f = \frac{(2.54533 \text{ cm})[1 - (23 \times 10^{-6} / \text{C}^0)(100 \degree C)] - (2.54000 \text{ cm})[1 - (17 \times 10^{-6} / \text{C}^0)(0 \degree C)]}{(2.54000 \text{ cm})(17 \times 10^{-6} / \text{C}^0) - (2.54533 \text{ cm})(23 \times 10^{-6} / \text{C}^0)}, \]
\[ = 34.1 \degree C. \]

No work is done, so we only have the issue of heat flow, then
\[ Q_r + Q_s = 0. \]
Where "r" refers to the copper ring and "s" refers to the aluminum sphere. The heat equations are
\[ Q_r = m_r c_r (T_f - T_{i,r}), \]
\[ Q_s = m_s c_s (T_f - T_{i,s}). \]
Equating and rearranging,
\[ m_s = m_r c_r (T_{i,r} - T_f) \]
\[ c_s (T_f - T_{i,s}) \]
or
\[ m_s = \frac{(21.6 \text{ g})(387 \text{ J/kg-K})(0 \degree C - 34.1 \degree C)}{(900 \text{ J/kg-K})(34.1 \degree C - 100 \degree C)} = 4.81 \text{ g}. \]
E23-30 Air is mostly diatomic (N₂ and O₂), so use γ = 1.4.

(a) \( pV^γ \) is a constant, so

\[
V_2 = V_1 \sqrt[γ]{\frac{p_1}{p_2}} = V_1 \sqrt[1.4]{\frac{1.0 \text{ atm}}{2.3 \text{ atm}}} = 0.552V_1.
\]

\[T_2 = T_1 \frac{p_2}{p_1} \frac{V_2}{V_1},\]

so

\[
T_2 = (291 \text{ K}) \frac{(2.3 \text{ atm}) (0.552V_1)}{(1.0 \text{ atm}) V_1} = 369 \text{ K},
\]

or 96°C.

(b) The work required for delivering 1 liter of compressed air is

\[
W_{12} = \frac{(1.01 \times 10^5 \text{ Pa/atm})(1 \times 10^{-3} \text{ m}^3/\text{l})}{(1.40) - 1} \left[ (2.3 \text{ atm})(1.0 \text{ l}) - (1.0 \text{ atm})(1.0 \text{ l}/0.552) \right] = 123 \text{ J}.
\]

The number of liters per second that can be delivered is then

\[
\Delta V/\Delta t = (230 \text{ W})/(123 \text{ J/l}) = 1.87 \text{ l/s}.
\]

E23-31 \( E_{\text{int,rot}} = nRT = (1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(298 \text{ K}) = 2480 \text{ J}.\)

E23-32 \( E_{\text{int,rot}} = \frac{3}{2}nRT = (1.5)(1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(523 \text{ K}) = 6520 \text{ J}.\)

E23-33 (a) Invert Eq. 32-20,

\[
\gamma = \frac{\ln(p_1/p_2)}{\ln(V_2/V_1)} = \frac{\ln(122 \text{ kPa/1450 kPa})}{\ln(1.36 \text{ m}^3/10.7 \text{ m}^3)} = 1.20.
\]

(b) The final temperature is found from the ideal gas law,

\[
T_f = T_1 \frac{p_fV_f}{p_1V_1} = (250 \text{ K}) \frac{(1450 \times 10^3 \text{ Pa})(1.36 \text{ m}^3)}{(122 \times 10^3 \text{ Pa})(10.7 \text{ m}^3)} = 378 \text{ K},
\]

which is the same as 105°C.

(c) Ideal gas law, again:

\[
n = \frac{pV}{RT} = \frac{[(1450 \times 10^3 \text{ Pa})(1.36 \text{ m}^3)]}{[(8.31 \text{ J/mol} \cdot \text{K})(378 \text{ K})]} = 628 \text{ mol}.
\]

(d) From Eq. 23-24,

\[
E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(628 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(250 \text{ K}) = 1.96 \times 10^6 \text{ J}
\]

before the compression and

\[
E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(628 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(378 \text{ K}) = 2.96 \times 10^6 \text{ J}
\]

after the compression.

(e) The ratio of the rms speeds will be proportional to the square root of the ratio of the internal energies,

\[
\sqrt{\frac{(1.96 \times 10^6 \text{ J})}{(2.96 \times 10^6 \text{ J})}} = 0.813;
\]

we can do this because the number of particles is the same before and after, hence the ratio of the energies per particle is the same as the ratio of the total energies.
E23-34 We can assume neon is an ideal gas. Then \( \Delta T = 2\Delta E_{\text{int}}/3nR \), or

\[
\Delta T = \frac{2(1.34 \times 10^{12} \text{eV})(1.6 \times 10^{-19} \text{J/eV})}{3(0.120 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})} = 1.43 \times 10^{-7} \text{J}.
\]

E23-35 At constant pressure, doubling the volume is the same as doubling the temperature. Then

\[
Q = nC_p \Delta T = (1.35 \text{ mol}) \frac{7}{2}(8.31 \text{ J/mol} \cdot \text{K})(568 \text{ K} - 284 \text{ K}) = 1.12 \times 10^4 \text{J}.
\]

E23-36 (a) \( n = m/M = (12 \text{ g})/(28 \text{ g/mol}) = 0.429 \text{ mol} \).

(b) This is a constant volume process, so

\[
Q = nC_V \Delta T = (0.429 \text{ mol}) \frac{5}{2}(8.31 \text{ J/mol} \cdot \text{K})(125^\circ \text{C} - 25^\circ \text{C}) = 891 \text{J}.
\]

E23-37 (a) From Eq. 23-37,

\[
Q = nc_p \Delta T = (4.34 \text{ mol})(29.1 \text{ J/mol} \cdot \text{K})(62.4 \text{ K}) = 7880 \text{J}.
\]

(b) From Eq. 23-28,

\[
E_{\text{int}} = \frac{5}{2}nR\Delta T = \frac{5}{2}(4.34 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(62.4 \text{ K}) = 5630 \text{J}.
\]

(c) From Eq. 23-23,

\[
K_{\text{trans}} = \frac{3}{2}nR\Delta T = \frac{3}{2}(4.34 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(62.4 \text{ K}) = 3380 \text{J}.
\]

E23-38 \( c_V = \frac{3}{2}(8.31 \text{ J/mol} \cdot \text{K})/(4.00 \text{ g/mol}) = 3120 \text{ J/kg} \cdot \text{K} \).

E23-39 Each species will experience the same temperature change, so

\[
Q = Q_1 + Q_2 + Q_3,
\]

\[
= n_1C_1\Delta T + n_2C_2\Delta T + n_3C_3\Delta T.
\]

Dividing this by \( n = n_1 + n_2 + n_3 \) and \( \Delta T \) will return the specific heat capacity of the mixture, so

\[
C = \frac{n_1C_1 + n_2C_2 + n_3C_3}{n_1 + n_2 + n_3}.
\]

E23-40 \( W_{AB} = 0 \), since it is a constant volume process, consequently, \( W = W_{AB} + W_{ABC} = -15 \text{J} \). But around a closed path \( Q = -W \), so \( Q = 15 \text{J} \). Then

\[
Q_{CA} = Q - Q_{AB} - Q_{BC} = (15 \text{J}) - (20 \text{J}) - (0 \text{J}) = -5 \text{J}.
\]

Note that this heat is removed from the system!
P23-2  (a) $H = (428 \text{ W/m} \cdot \text{K})(4.76 \times 10^{-4} \text{ m}^2)(100 \text{ C}^\circ)/(1.17 \text{ m}) = 17.4 \text{ W}$.  
(b) $\Delta m/\Delta t = H/L = (17.4 \text{ W})/(333 \times 10^3 \text{ J/kg}) = 5.23 \times 10^{-6} \text{ kg/s}$, which is the same as 188 g/h.

P23-3  Follow the example in Sample Problem 23-2. We start with Eq. 23-1:

$$H = kA \frac{dT}{dr},$$

$$H = k(4\pi r^2) \frac{dT}{dr},$$

$$\int_{r_1}^{r_2} H \frac{dr}{4\pi r^2} = \int_{T_1}^{T_2} kdT,$$

$$\frac{H}{4\pi} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) = k(T_1 - T_2),$$

$$H \left( \frac{r_2 - r_1}{r_1 r_2} \right) = 4\pi k(T_1 - T_2),$$

$$H = \frac{4\pi k(T_1 - T_2) r_1 r_2}{r_2 - r_1}.$$

P23-4  (a) $H = (54 \times 10^{-3} \text{ W/m}^2)4\pi(6.37 \times 10^6 \text{ m})^2 = 2.8 \times 10^{13} \text{ W}$.  
(b) Using the results of Problem 23-3,

$$\Delta T = \frac{(2.8 \times 10^{13} \text{ W})(6.37 \times 10^6 \text{ m} - 3.47 \times 10^6 \text{ m})}{4\pi(4.2 \text{ W/m} \cdot \text{K})(6.37 \times 10^6 \text{ m})(3.47 \times 10^6 \text{ m})} = 7.0 \times 10^4 \text{ C}^\circ.$$  
Since $T_2 = 0 \text{ C}^\circ$, we expect $T_1 = 7.0 \times 10^4 \text{ C}^\circ$.

P23-5  Since $H = -kA dT/dr$, then $H \, dx = -aT \, dT$. $H$ is a constant, so integrate both side according to

$$\int H \, dx = - \int aT \, dT,$$

$$HL = -\frac{1}{2}(T_2^2 - T_1^2),$$

$$H = \frac{aA}{2L}(T_2^2 - T_1^2).$$

P23-6  Assume the water is all at $0 \text{ C}^\circ$. The heat flow through the ice is then $H = kA\Delta T/x$; the rate of ice formation is $\Delta m/\Delta t = H/L$. But $\Delta m = \rho A \Delta x$, so

$$\frac{\Delta x}{\Delta t} = \frac{H}{\rho AL} = \frac{k\Delta T}{\rho Lx},$$

$$\frac{(1.7 \text{ W/m} \cdot \text{K})(10 \text{ C}^\circ)}{(920 \text{ kg/m}^3)(333 \times 10^2 \text{ J/kg})(0.05 \text{ m})} = 1.11 \times 10^{-6} \text{ m/s}.$$

That's the same as 0.40 cm/h.

P23-7  (a) Start with the heat equation:

$$Q_t + Q_i + Q_w = 0,$$
where \( Q_t \) is the heat from the tea, \( Q_i \) is the heat from the ice when it melts, and \( Q_w \) is the heat from the water (which used to be ice). Then

\[
m_t c_t (T_f - T_{t,i}) + m_i L_f + m_w c_w (T_f - T_{w,i}) = 0,
\]

which, since we have assumed all of the ice melts and the masses are all equal, can be solved for \( T_f \) as

\[
T_f = \frac{c_t T_{t,i} + c_w T_{w,i} - L_f}{c_t + c_w},
\]

\[
= \frac{(4190 \text{ J/kg} \cdot \text{K})(90^\circ \text{C}) + (4190 \text{ J/kg} \cdot \text{K})(0^\circ \text{C}) - (333 \times 10^3 \text{ J/kg})}{(4190 \text{ J/kg} \cdot \text{K}) + (4190 \text{ J/kg} \cdot \text{K})},
\]

\[
= 5.3^\circ \text{C}.
\]

(b) Once again, assume all of the ice melted. Then we can do the same steps, and we get

\[
T_f = \frac{c_t T_{t,i} + c_w T_{w,i} - L_f}{c_t + c_w},
\]

\[
= \frac{(4190 \text{ J/kg} \cdot \text{K})(70^\circ \text{C}) + (4190 \text{ J/kg} \cdot \text{K})(0^\circ \text{C}) - (333 \times 10^3 \text{ J/kg})}{(4190 \text{ J/kg} \cdot \text{K}) + (4190 \text{ J/kg} \cdot \text{K})},
\]

\[
= -4.7^\circ \text{C}.
\]

So we must have guessed wrong when we assumed that all of the ice melted. The heat equation then simplifies to

\[
m_t c_t (T_f - T_{t,i}) + m_i L_f = 0,
\]

and then

\[
m_i = \frac{m_t c_t (T_{t,i} - T_f)}{L_f},
\]

\[
= \frac{(0.520 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(90^\circ \text{C} - 0^\circ \text{C})}{(333 \times 10^3 \text{ J/kg})},
\]

\[
= 0.458 \text{ kg}.
\]

P23-8 \( c = \frac{Q}{m \Delta T} = H/(\Delta m/\Delta t) \Delta T \). But \( \Delta m/\Delta t = \rho \Delta V/\Delta t \). Combining,

\[
c = \frac{H}{(\Delta V/\Delta t) \rho \Delta T} = \frac{(250 \text{ W})}{(8.2 \times 10^{-6} \text{ m}^3/\text{s})(0.85 \times 10^3 \text{ kg/m}^3)(15 \text{ C}^\circ)} = 2.4 \times 10^2 \text{ J/kg} \cdot \text{ K}.
\]

P23-9 (a) \( n = \frac{N_A}{M} \), so

\[
\epsilon = \frac{(2256 \times 10^3 \text{ J/kg})}{(6.02 \times 10^{23} / \text{mol})/(0.018 \text{ kg/mol})} = 6.75 \times 10^{-20} \text{ J}.
\]

(b) \( E_{av} = \frac{1}{2} kT \), so

\[
\epsilon = \frac{2(6.75 \times 10^{-20} \text{ J})}{3(1.38 \times 10^{-23} \text{ J/K})(305 \text{ K})} = 10.7.
\]

P23-10 \( Q_w + Q_t = 0 \), so

\[
C_t \Delta T_t + m_w c_w (T_f - T_i) = 0,
\]

or

\[
T_i = \frac{(0.3 \text{ kg})(4190 \text{ J/kg} \cdot \text{m})(44.4^\circ \text{C}) + (46.1 \text{ J/K})(44.4^\circ \text{C} - 15.0^\circ \text{C})}{(0.3 \text{ kg})(4190 \text{ J/kg} \cdot \text{m})} = 45.5^\circ \text{C}.
\]
We can use Eq. 23-10, but we will need to approximate \( c \) first. If we assume that the line is straight then we use \( c = nT + b \). I approximate \( m \) from

\[
m = \frac{14 \text{ J/mol} \cdot \text{K}}{500 \text{ K}} - \frac{3 \text{ J/mol} \cdot \text{K}}{200 \text{ K}} = 3.67 \times 10^{-2} \text{ J/mol}.
\]

Then I find \( b \) from those same data points,

\[
b = (3 \text{ J/mol} \cdot \text{K}) - (3.67 \times 10^{-2} \text{ J/mol})(200 \text{ K}) = -4.34 \text{ J/mol} \cdot \text{K}.
\]

Then from Eq. 23-10,

\[
Q = n \int_{T_1}^{T_f} c \, dT,
\]

\[
= n \int_{T_1}^{T_f} (mT + b) \, dT,
\]

\[
= n \left[ \frac{m}{2} T^2 + bT \right]_{T_1}^{T_f},
\]

\[
= n \left( \frac{m}{2} (T_f^2 - T_1^2) + b(T_f - T_1) \right),
\]

\[
= (0.45 \text{ mol}) \left( \frac{3.67 \times 10^{-2} \text{ J/mol}}{2} \right) \frac{(500 \text{ K})^2 - (200 \text{ K})^2}{(500 \text{ K} - 200 \text{ K})},
\]

\[
= 1.15 \times 10^3 \text{ J}.
\]

\[
\delta Q = nC \delta T, \text{ so}
\]

\[
Q = n \int C \, dT,
\]

\[
= n \left[ (0.318 \text{ J/mol} \cdot \text{K})T^2/2 - (0.00109 \text{ J/mol} \cdot \text{K}^3)T^3/3 - (0.628 \text{ J/mol} \cdot \text{K})T \right]_{50 \text{ K}}^{90 \text{ K}},
\]

\[
= n(645.8 \text{ J/mol}).
\]

Finally,

\[
Q = (645.8 \text{ J/mol})(316 \text{ g})/(107.87 \text{ g/mol}) = 189 \text{ J}.
\]

\[TV^{7-1} \text{ is a constant, so}
\]

\[
T_2 = (292 \text{ K})(1/1.28)^{(1.40)-1} = 265 \text{ K}
\]

\[W = -\int p \, dV, \text{ so}
\]

\[
W = -\int \left[ \frac{nRT}{V - nb} - \frac{an^2}{V^2} \right] \, dV,
\]

\[
= -nRT \ln(V - nb) - \frac{an^2}{V} \Big|_i^f,
\]

\[
= -nRT \ln \frac{V_f - nb}{V_i - nb} - an^2 \left( \frac{1}{V_f} - \frac{1}{V_i} \right).
\]
When the tube is horizontal there are two regions filled with gas, one at $p_{1,i}$, $V_{1,i}$; the other at $p_{2,i}$, $V_{2,i}$. Originally $p_{1,i} = p_{2,i} = 1.01 \times 10^6 \text{Pa}$ and $V_{1,i} = V_{2,i} = (0.45 \text{ m})A$, where $A$ is the cross sectional area of the tube.

When the tube is held so that region 1 is on top then the mercury has three forces on it: the force of gravity, $mg$; the force from the gas above pushing down $p_{2,f}A$; and the force from the gas below pushing up $p_{1,f}A$. The balanced force expression is

$$p_{1,f}A = p_{2,f}A + mg.$$ 

If we write $m = \rho l_m A$ where $l_m = 0.10 \text{ m}$, then

$$p_{1,f} = p_{2,f} + \rho g l_m.$$ 

Finally, since the tube has uniform cross section, we can write $V = Al$ everywhere.

(a) For an isothermal process $p_1 l_i = p_2 l_f$, where we have used $V = Al$, and then

$$p_{1,i} \frac{l_{1,i}}{l_{1,f}} - p_{2,i} \frac{l_{2,i}}{l_{2,f}} = \rho g l_m.$$ 

But we can factor out $p_{1,i} = p_{2,i}$ and $l_{1,i} = l_{2,i}$, and we can apply $l_{1,f} + l_{2,f} = 0.90 \text{ m}$. Then

$$\frac{1}{l_{1,f}} - \frac{1}{0.90 \text{ m} - l_{1,f}} = \frac{\rho g l_m}{p l_i}.$$ 

Put in some numbers and rearrange,

$$0.90 \text{ m} - 2l_{1,f} = (0.294 \text{ m}^{-1})l_{1,f}(0.90 \text{ m} - l_{1,f}),$$

which can be written as an ordinary quadratic,

$$(0.294 \text{ m}^{-1})l_{1,f}^2 - (2.265)l_{1,f} + (0.90 \text{ m}) = 0$$

The solutions are $l_{1,f} = 7.284 \text{ m}$ and $0.421 \text{ m}$. Only one of these solutions is reasonable, so the mercury shifted down $0.450 - 0.421 = 0.029 \text{ m}$.

(b) The math is a wee bit uglier here, but we can start with $p_1 l_i^\gamma = p_2 l_f^\gamma$, and this means that everywhere we had a $l_{1,f}$ in the previous derivation we need to replace it with $l_{1,f}^\gamma$. Then we have

$$\frac{1}{l_{1,f}^\gamma} - \frac{1}{(0.90 \text{ m} - l_{1,f})^\gamma} = \frac{\rho g l_m}{p l_i^\gamma}.$$ 

This can be written as

$$(0.90 \text{ m} - l_{1,f})^\gamma - l_{1,f}^\gamma = (0.404 \text{ m}^{-\gamma})l_{1,f}^\gamma(0.90 \text{ m} - l_{0.1,f})^\gamma,$$

which looks nasty to me! I'll use Maple to get the answer, and find $l_{1,f} = 0.429$, so the mercury shifted down $0.450 - 0.429 = 0.021 \text{ m}$.

Which is more likely? Turn the tube fast, and the adiabatic approximation works. Eventually the system will return to room temperature, and then the isothermal approximation is valid.

Internal energy for an ideal diatomic gas can be written as

$$E_{\text{int}} = \frac{5}{2} nRT = \frac{5}{2} pV,$$

simply by applying the ideal gas law. The room, however, has a fixed pressure and volume, so the internal energy is independent of the temperature. As such, any energy supplied by the furnace leaves the room, either as heat or as expanding gas doing work on the outside.
E24-7 Use the first equation on page 551.

\[ n = \frac{\Delta S}{R \ln(V_f/V_i)} = \frac{(24 \text{ J/K})}{(8.31 \text{ J/mol} \cdot \text{K}) \ln(3.4/1.3)} = 3.00 \text{ mol.} \]

E24-8 \( \Delta S = Q/T_c - Q/T_h \).

(a) \( \Delta S = (260 \text{ J})(1/100 \text{ K} - 1/400 \text{ K}) = 1.95 \text{ J/K} \).
(b) \( \Delta S = (260 \text{ J})(1/200 \text{ K} - 1/400 \text{ K}) = 0.65 \text{ J/K} \).
(c) \( \Delta S = (260 \text{ J})(1/300 \text{ K} - 1/400 \text{ K}) = 0.217 \text{ J/K} \).
(d) \( \Delta S = (260 \text{ J})(1/360 \text{ K} - 1/400 \text{ K}) = 0.0722 \text{ J/K} \).

E24-9 (a) If the rod is in a steady state we wouldn't expect the entropy of the rod to change. Heat energy is flowing out of the hot reservoir into the rod, but this process happens at a fixed temperature, so the entropy change in the hot reservoir is

\[ \Delta S_H = \frac{Q_H}{T_H} = \frac{(-1200 \text{ J})}{(403 \text{ K})} = -2.98 \text{ J/K}. \]

The heat energy flows into the cold reservoir, so

\[ \Delta S_C = \frac{Q_H}{T_H} = \frac{(1200 \text{ J})}{(297 \text{ K})} = 4.04 \text{ J/K}. \]

The total change in entropy of the system is the sum of these two terms

\[ \Delta S = \Delta S_H + \Delta S_C = 1.06 \text{ J/K}. \]

(b) Since the rod is in a steady state, nothing is changing, not even the entropy.

E24-10 (a) \( Q_c + Q_i = 0 \), so

\[ m_c c_c(T - T_c) + m_i c_i(T - T_i) = 0, \]

which can be solved for \( T \) to give

\[ T = \frac{(0.05 \text{ kg})(387 \text{ J/kg} \cdot \text{K})(400 \text{ K}) + (0.10 \text{ kg})(129 \text{ J/kg} \cdot \text{K})(200 \text{ K})}{(0.05 \text{ kg})(387 \text{ J/kg} \cdot \text{K}) + (0.10 \text{ kg})(129 \text{ J/kg} \cdot \text{K})} = 320 \text{ K}. \]

(b) Zero.
(c) \( \Delta S = m_c c_c(T_f/T_i) \), so

\[ \Delta S = (0.05 \text{ kg})(387 \text{ J/kg} \cdot \text{K}) \ln \left( \frac{320 \text{ K}}{400 \text{ K}} \right) + (0.10 \text{ kg})(129 \text{ J/kg} \cdot \text{K}) \ln \left( \frac{320 \text{ K}}{200 \text{ K}} \right) = 1.75 \text{ J/K}. \]

E24-11 The total mass of ice and water is 2.04 kg. If eventually the ice and water have the same mass, then the final state will have 1.02 kg of each. This means that 1.78 kg - 1.02 kg = 0.76 kg of water changed into ice.

(a) The change of water at 0°C to ice at 0°C is isothermal, so the entropy change is

\[ \Delta S = \frac{Q}{T} = -\\frac{mL}{T} = \frac{(0.76 \text{ kg})(333 \times 10^3 \text{ J/kg})}{(273 \text{ K})} = -927 \text{ J/K}. \]

(b) The entropy change is now +927 J/K.
E24-12  (a) \( Q_a + Q_w = 0 \), so
\[
m_a c_a (T - T_a) + m_w c_w (T - T_w) = 0,
\]
which can be solved for \( T \) to give
\[
T = \frac{(0.196 \text{ kg})(900 \text{ J/kg·K})(380 \text{ K}) + (0.0523 \text{ kg})(4190 \text{ J/kg·K})(292 \text{ K})}{(0.196 \text{ kg})(900 \text{ J/kg·K}) + (0.0523 \text{ kg})(4190 \text{ J/kg·K})} = 331 \text{ K}.
\]
That's the same as 58°C.
(b) \( \Delta S = mc \ln \frac{T_f}{T_i} \), so
\[
\Delta S_a = (0.196 \text{ kg})(900 \text{ J/kg·K}) \ln \frac{(331 \text{ K})}{(380 \text{ K})} = -24.4 \text{ J/K}.
\]
(c) For the water,
\[
(0.0523 \text{ kg})(4190 \text{ J/kg·K}) \ln \frac{(331 \text{ K})}{(292 \text{ K})} = 27.5 \text{ J/K}.
\]
(d) \( \Delta S = (27.5 \text{ J/K}) + (-24.4 \text{ J/K}) = 3.1 \text{ J/K} \).

E24-13  (a) \( e = 1 - (36.2 \text{ J/52.4 J}) = 0.309 \).
(b) \( W = Q_h - Q_c = (52.4 \text{ J}) - (36.2 \text{ J}) = 16.2 \text{ J} \).

E24-14  (a) \( Q_h = (8.18 \text{ kJ})/(0.25) = 32.7 \text{ kJ}, \ Q_c = Q_h - W = (32.7 \text{ kJ}) - (8.18 \text{ kJ}) = 24.5 \text{ kJ} \).
(b) \( Q_h = (8.18 \text{ kJ})/(0.31) = 26.4 \text{ kJ}, \ Q_c = Q_h - W = (26.4 \text{ kJ}) - (8.18 \text{ kJ}) = 18.2 \text{ kJ} \).

E24-15  One hour's worth of coal, when burned, will provide energy equal to
\[
(382 \times 10^3 \text{ kg})(28.0 \times 10^6 \text{ J/kg}) = 1.07 \times 10^{13} \text{ J}.
\]
In this hour, however, the plant only generates
\[
(755 \times 10^6 \text{ W})(3600 \text{ s}) = 2.72 \times 10^{12} \text{ J}.
\]
The efficiency is then
\[
e = \left(\frac{2.72 \times 10^{12} \text{ J}}{1.07 \times 10^{13} \text{ J}}\right) = 25.4\%.
\]

E24-16  We use the convention that all quantities are positive, regardless of direction. \( W_A = 5W_B;\)
\( Q_{i,A} = 3Q_{i,B}; \) and \( Q_{o,A} = 2Q_{o,B}. \) But \( W_A = Q_{i,A} - Q_{o,A}, \) so
\[
5W_B = 3Q_{i,B} - 2Q_{o,B},
\]
or, applying \( W_B = Q_{i,B} - Q_{o,B}, \)
\[
5W_B = 3Q_{i,B} - 2(Q_{i,B} - W_B),
3W_B = Q_{i,B},
W_B/Q_{i,B} = 1/3 = e_B.
\]
Then
\[
e_A = \frac{W_A}{Q_{i,A}} = \frac{5W_B}{3Q_{i,B}} = \frac{5}{3} = \frac{5}{9}.
\]

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We want to evaluate:

\[
\Delta S = \int_{T_1}^{T_f} \frac{n \gamma \Delta V}{T} \, dT,
\]

\[
= \int_{T_1}^{T_f} \frac{nAT^3}{T} \, dT,
\]

\[
= \int_{T_1}^{T_f} nAT^2 \, dT,
\]

\[
= \frac{nA}{3} (T_f^3 - T_1^3).
\]

Into this last expression, which is true for many substances at sufficiently low temperatures, we substitute the given numbers.

\[
\Delta S = \frac{(4.8 \text{ mol})(3.15 \times 10^{-5} \text{ J/mol} \cdot \text{K}^4)}{3} \left((10 \text{ K})^3 - (5.0 \text{ K})^3\right) = 4.41 \times 10^{-2} \text{ J/K}.
\]

(b) \(\Delta E_{fbc} = \frac{3}{2} nR \Delta T_{bc}\), with a little algebra,

\[
\Delta E_{fbc} = \frac{3}{2} nRT_c - nRT_b = \frac{3}{2} (p_cV_c - p_bV_b) = \frac{3}{2} (8 - 4)p_0V_0 = 6p_0V_0.
\]

\(\Delta S_{bc} = \frac{3}{2} nR \ln(T_c/T_b)\), with a little algebra,

\[
\Delta S_{bc} = \frac{3}{2} nR \ln(p_c/p_b) = \frac{3}{2} nR \ln 2.
\]

(c) Both are zero for a cyclic process.

(a) For an isothermal process,

\[p_2 = p_1(V_1/V_2) = p_1/3.\]

For an adiabatic process,

\[p_3 = p_1(V_1/V_2)^\gamma = p_1(1/3)^{1.4} = 0.215p_1.\]

For a constant volume process,

\[T_3 = T_2(p_3/p_2) = T_1(0.215/0.333) = 0.646T_1.\]

(b) The easiest ones first: \(\Delta E_{f12} = 0, W_{23} = 0, Q_{31} = 0, \Delta S_{31} = 0\). The next easier ones:

\[
\Delta E_{f23} = \frac{5}{2} nR \Delta T_{23} = \frac{5}{2} nR(0.646T_1 - T_1) = -0.885p_1V_1,
\]

\[
Q_{23} = \Delta E_{f23} - W_{23} = -0.885p_1V_1,
\]

\[
\Delta E_{f31} = -\Delta E_{f23} - \Delta E_{f12} = 0.885p_1V_1,
\]

\[
W_{31} = \Delta E_{f31} - Q_{31} = 0.885p_1V_1.
\]
Finally, some harder ones:

\[ W_{12} = -nRT_1 \ln(V_2/V_1) = -p_1 V_1 \ln(3) = -1.10p_1 V_1, \]

\[ Q_{12} = \Delta E_{\text{int}12} - W_{12} = 1.10p_1 V_1. \]

And now, the hardest:

\[ \Delta S_{12} = Q_{12} / T_1 = 1.10nR, \]

\[ \Delta S_{23} = -\Delta S_{12} - \Delta S_{31} = -1.10nR. \]

P24-5  Note that \( T_A = T_B = T_C / 4 = T_D \).

Process I: \( ABC \)

(a) \( Q_{AB} = -W_{AB} = nRT_0 \ln(V_B/V_A) = p_0 V_0 \ln 2 \). \( Q_{BC} = \frac{3}{2} nR(T_C - T_B) = \frac{3}{2}(p_C V_C - p_B V_B) = \frac{3}{2}(4p_0 V_0 - p_0 V_0) = 4.5p_0 V_0. \)

(b) \( W_{AB} = -nRT_0 \ln(V_B/V_A) = -p_0 V_0 \ln 2 \). \( W_{BC} = 0. \)

(c) \( E_{\text{int}} = \frac{3}{2} nR(T_C - T_A) = \frac{3}{2}(p_C V_C - p_A V_A) = \frac{3}{2}(4p_0 V_0 - p_0 V_0) = 4.5p_0 V_0. \)

(d) \( \Delta S_{AB} = nR \ln(V_B/V_A) = nR \ln 2 \); \( \Delta S_{BC} = \frac{3}{2} nR \ln(T_C/T_B) = \frac{3}{2} nR \ln 4 = 3nR \ln 2. \) Then \( \Delta S_{AC} = 4nR. \)

Process II: \( ADC \)

(a) \( Q_{AD} = -W_{AD} = nRT_0 \ln(V_D/V_A) = p_0 V_0 \ln 2 \). \( Q_{DC} = \frac{3}{2} nR(T_C - T_D) = \frac{3}{2}(p_C V_C - p_D V_D) = \frac{3}{2}(4p_0 V_0 - p_0 V_0) = 10p_0 V_0. \)

(b) \( W_{AB} = -nRT_0 \ln(V_D/V_A) = p_0 V_0 \ln 2 \). \( W_{DC} = -p_0 V_0 = -p_0 (2V_0 - V_0/2) = -\frac{3}{2} p_0 V_0. \)

(c) \( E_{\text{int}} = \frac{3}{2} nR(T_C - T_A) = \frac{3}{2}(p_C V_C - p_A V_A) = \frac{3}{2}(4p_0 V_0 - p_0 V_0) = 4.5p_0 V_0. \)

(d) \( \Delta S_{AD} = nR \ln(V_D/V_A) = -nR \ln 2 \); \( \Delta S_{DC} = \frac{3}{2} nR \ln(T_C/T_D) = \frac{3}{2} nR \ln 4 = 5nR \ln 2. \) Then \( \Delta S_{AC} = 4nR. \)

P24-6  The heat required to melt the ice is

\[ Q = m(c_w \Delta T_{23} + L + c_s \Delta T_{12}), \]

\[ = (0.0126 \text{ kg})[(4190 \text{ J/kg} \cdot \text{ K})(15 \text{ C})] + (333 \times 10^3 \text{ J/kg}) + (2220 \text{ J/kg} \cdot \text{ K})(10 \text{ C}), \]

\[ = 5270 \text{ J}. \]

The change in entropy of the ice is

\[ \Delta S_1 = m[c_w \ln(T_3/T_2) + L/T_2 + c_s \ln(T_2/T_1)], \]

\[ = (0.0126 \text{ kg})[(4190 \text{ J/kg} \cdot \text{ K}) \ln(288/273) + (333 \times 10^3 \text{ J/kg})/(273 \text{ K}), \]

\[ + (2220 \text{ J/kg} \cdot \text{ K}) \ln(273/263)], \]

\[ = 19.24 \text{ J/K}. \]

The change in entropy of the lake is \( \Delta S_1 = (5270 \text{ J})/(288 \text{ K}) = 18.29 \text{ J/K}. \) The change in entropy of the system is 0.95 J/kg.

P24-7  (a) This is a problem where the total internal energy of the two objects doesn't change, but since no work is done during the process, we can start with the simpler expression \( Q_1 + Q_2 = 0 \).

The heat transfers by the two objects are

\[ Q_1 = m_1 c_1 (T_1 - T_{1,i}), \]

\[ Q_2 = m_2 c_2 (T_2 - T_{2,i}). \]

Note that we don't call the final temperature \( T_f \) here, because we are not assuming that the two objects are at equilibrium.
We combine these three equations,

\[ m_2 c_2 (T_2 - T_{2,1}) = -m_1 c_1 (T_1 - T_{1,1}) , \]
\[ m_2 c_2 T_2 = m_2 c_2 T_{2,1} + m_1 c_1 (T_{1,1} - T_1) , \]
\[ T_2 = T_{2,1} + \frac{m_1 c_1}{m_2 c_2} (T_{1,1} - T_1) . \]

As object 1 "cools down", object 2 "heats up", as expected.

(b) The entropy change of one object is given by

\[ \Delta S = \int_{T_1}^{T_f} \frac{mc dT}{T} = mc \ln \frac{T_f}{T_1} , \]

and the total entropy change for the system will be the sum of the changes for each object, so

\[ \Delta S = m_1 c_1 \ln \frac{T_1}{T_{1,1}} + m_2 c_2 \ln \frac{T_2}{T_{1,2}} . \]

Into the this last equation we need to substitute the expression for \( T_2 \) in as a function of \( T_1 \). There’s no new physics in doing this, just a mess of algebra.

(c) We want to evaluate \( d(\Delta S)/dT_1 \). To save on algebra we will work with the last expression, remembering that \( T_2 \) is a function, not a variable. Then

\[ \frac{d(\Delta S)}{dT_1} = \frac{m_1 c_1}{T_1} + \frac{m_2 c_2}{T_2} \frac{dT_2}{dT_1} . \]

We’ve saved on algebra, but now we need to evaluate \( dT_2/dT_1 \). Starting with the results from part (a),

\[ \frac{dT_2}{dT_1} = \frac{d}{dT_1} \left( T_{2,1} + \frac{m_1 c_1}{m_2 c_2} (T_{1,1} - T_1) \right) , \]
\[ = -\frac{m_1 c_1}{m_2 c_2} . \]

Now we collect the two results and write

\[ \frac{d(\Delta S)}{dT_1} = \frac{m_1 c_1}{T_1} + \frac{m_2 c_2}{T_2} \left( \frac{m_1 c_1}{m_2 c_2} \right) , \]
\[ = m_1 c_1 \left( \frac{1}{T_1} - \frac{1}{T_2} \right) . \]

We could consider writing \( T_2 \) out in all of its glory, but what would it gain us? Nothing. There is actually considerably more physics in the expression as written, because...

(d) ...we get a maximum for \( \Delta S \) when \( d(\Delta S)/dT_1 = 0 \), and this can only occur when \( T_1 = T_2 \) according to the expression.

P24-8 \( T_b = (10.4 \times 1.01 \times 10^5 \text{ Pa})(1.22 \text{ m}^3)/(2 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K}) = 7.71 \times 10^4 \text{ K} \). Maybe not so realistic? \( T_a \) can be found after finding

\[ p_c = p_b (V_b/V_c)^7 = (10.4 \times 1.01 \times 10^5 \text{ Pa})(1.22/9.13)^{1.67} = 3.64 \times 10^4 \text{ Pa} , \]

Then

\[ T_a = T_b (p_a/p_b) = (7.71 \times 10^4 \text{ K})(3.64 \times 10^4/1.05 \times 10^5) = 2.67 \times 10^3 \text{ K} . \]
Similarly, \( T_c = T_a(V_c/V_a) = (2.67 \times 10^5 \text{K})(9.13/1.22) = 2.00 \times 10^4 \text{K} \).

(a) Heat is added during process ab only;
\[ Q_{ab} = \frac{3}{2} nR(T_b - T_a) = \frac{3}{2} (2 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(7.71 \times 10^4 \text{K} - 2.67 \times 10^5 \text{K}) = 1.85 \times 10^6 \text{J} \]

(b) Heat is removed during process ca only;
\[ Q_{ca} = \frac{5}{2} nR(T_a - T_c) = \frac{5}{2} (2 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(2.67 \times 10^5 \text{K} - 2.00 \times 10^4 \text{K}) = -0.721 \times 10^6 \text{J} \]

(c) \( W = Q_{ab} - Q_{ca} = (1.85 \times 10^6 \text{J}) - (-0.721 \times 10^6 \text{J}) = 1.13 \times 10^6 \text{J} \).

(d) \( e = W/Q_{ab} = (1.13 \times 10^6)/(1.85 \times 10^6) = 0.611 \).

P24-9 The \( pV \) diagram for this process is Figure 23-21, except the cycle goes clockwise.

(a) Heat is input during the constant volume heating and the isothermal expansion. During heating,
\[ Q_1 = \frac{3}{2} nR\Delta T = \frac{3}{2} (1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{K} - 300 \text{K}) = 3740 \text{J} \]

During isothermal expansion,
\[ Q_2 = -W_2 = nRT \ln(V_f/V_i) = (1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{K}) \ln(2) = 3460 \text{J} \]

so \( Q_{in} = 7200 \text{J} \).

(b) Work is only done during the second and third processes; we've already solved the second,
\[ W_2 = -3460 \text{J} \]

\[ W_3 = -p\Delta V = p_aV_c - p_aV_a = nR(T_c - T_a) = (1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{K} - 300 \text{K}) = 2490 \text{J} \]

So \( W = -970 \text{J} \).

(c) \( e = |W|/|Q_{in}| = (970 \text{J})/(7200 \text{J}) = 0.13 \).

P24-10 (a) \( T_b = T_a(p_b/p_a) = 3T_a \);
\[ T_c = T_b(V_b/V_c)\gamma^{-1} = 3T_a(1/4)^{0.4} = 1.72T_a \]
\[ p_c = p_b(V_b/V_c)\gamma = 3p_a(1/4)^{1.4} = 0.43p_a \]
\[ T_d = T_a(V_d/V_a)\gamma^{-1} = T_a(1/4)^{0.4} = 0.57T_a \]
\[ p_d = p_a(V_d/V_a)\gamma = p_a(1/4)^{1.4} = 0.14p_a \]

(b) Heat in occurs during process ab, so \( Q_1 = \frac{5}{2} nR\Delta T_{ab} = 5nRT_a \); Heat out occurs during process cd, so \( Q_o = \frac{5}{2} nR\Delta T_{cd} = 2.87nRT_a \). Then
\[ e = 1 - (2.87nRT_a/5nRT_a) = 0.426 \]

P24-11 (c) \( (V_B/V_A) = (p_A/p_B) = (0/0.5) = 2 \). The work done on the gas during the isothermal compression is
\[ W = -nRT \ln(V_B/V_A) = -(1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(300 \text{K}) \ln(2) = -1730 \text{J} \]

Since \( \Delta E_{int} = 0 \) along an isotherm \( Q_b = 1730 \text{J} \).

The cycle has an efficiency of \( e = 1 - (100/300) = 2/3 \). Then for the cycle,
\[ W = eQ_b = (2/3)(1730 \text{J}) = 1150 \text{J} \].
E39-1 Both scales are logarithmic; choose any data point from the right hand side such as
\[ c = f \lambda \approx (1 \text{ Hz})(3 \times 10^4 \text{ m}) = 3 \times 10^8 \text{ m/s}, \]
and another from the left hand side such as
\[ c = f \lambda \approx (1 \times 10^{21} \text{ Hz})(3 \times 10^{-13} \text{ m}) = 3 \times 10^8 \text{ m/s}. \]

E39-2 (a) \( f = v/\lambda = (3.0 \times 10^8 \text{ m/s})/(1.0 \times 10^6)(6.37 \times 10^6 \text{ m}) = 4.7 \times 10^{-3} \text{ Hz}. \) If we assume that this is the data transmission rate in bits per second (a generous assumption), then it would take 140 days to download a web-page which would take only 1 second on a 56K modem!
(b) \( T = 1/f = 212 \text{s} = 3.5 \text{ min}. \)

E39-3 (a) Apply \( v = f\lambda. \) Then
\[ f = (3.0 \times 10^8 \text{ m/s})/(0.067 \times 10^{-15} \text{ m}) = 4.5 \times 10^{24} \text{ Hz}. \]
(b) \( \lambda = (3.0 \times 10^8 \text{ m/s})/(30 \text{ Hz}) = 1.0 \times 10^7 \text{ m}. \)

E39-4 Don't simply take reciprocal of linewidth! \( f = c/\lambda, \) so \( \delta f = (-c/\lambda^2)\delta \lambda. \) Ignore the negative, \( \delta f = (3.00 \times 10^8 \text{ m/s})(0.010 \times 10^{-9} \text{ m})/(632.8 \times 10^{-9} \text{ m})^2 = 7.5 \times 10^9 \text{ Hz}. \)

E39-5 (a) We refer to Fig. 39-6 to answer this question. The limits are approximately 520 nm and 620 nm.
(b) The wavelength for which the eye is most sensitive is 550 nm. This corresponds to a frequency of
\[ f = c/\lambda = (3.00 \times 10^8 \text{ m/s})(550 \times 10^{-9} \text{ m}) = 5.45 \times 10^{14} \text{ Hz}. \]
This frequency corresponds to a period of \( T = 1/f = 1.83 \times 10^{-15} \text{s}. \)

E39-6 \( f = c/\lambda. \) The number of complete pulses is \( ft, \) or
\[ ft = ct/\lambda = (3.00 \times 10^8 \text{ m/s})(430 \times 10^{-12} \text{s})/(520 \times 10^{-9} \text{ m}) = 2.48 \times 10^5. \]

E39-7 (a) \( 2(4.34 \text{ y}) = 8.68 \text{ y}. \)
(b) \( 2(2.2 \times 10^5 \text{ y}) = 4.4 \times 10^5 \text{ y}. \)

E39-8 (a) \( t = (150 \times 10^8 \text{ m})/(3 \times 10^8 \text{ m/s}) = 5 \times 10^{-4} \text{s}. \)
(b) The distance traveled by the light is \((1.5 \times 10^{11} \text{ m}) + 2(3.8 \times 10^8 \text{ m}), \) so
\[ t = (1.51 \times 10^{11} \text{ m})/(3 \times 10^8 \text{ m/s}) = 503 \text{s}. \]
(c) \( t = 2(1.3 \times 10^{12} \text{ m})/(3 \times 10^8 \text{ m/s}) = 8670 \text{s}. \)
(d) \( 1054 - 6500 \approx 5400 \text{ BC}. \)

E39-9 This is a question of how much time it takes light to travel 4 cm, because the light traveled from the Earth to the moon, bounced off of the reflector, and then traveled back. The time to travel 4 cm is \( \Delta t = (0.04 \text{ m})/(3 \times 10^8 \text{ m/s}) = 0.13 \text{ ns}. \) Note that I interpreted the question differently than the answer in the back of the book.
E39-10 Consider any incoming ray. The path of the ray can be projected onto the xy plane, the xz plane, or the yz plane. If the projected rays is exactly reflected in all three cases then the three dimensional incoming ray will be reflected exactly reversed. But the problem is symmetric, so it is sufficient to show that any plane works.

Now the problem has been reduced to Sample Problem 39-2, so we are done.

E39-11 We will choose the mirror to lie in the xy plane at z = 0. There is no loss of generality in doing so; we had to define our coordinate system somehow. The choice is convenient in that any normal is then parallel to the z axis. Furthermore, we can arbitrarily define the incident ray to originate at (0, 0, z₁). Lastly, we can rotate the coordinate system about the z axis so that the reflected ray passes through the point (0, y₂, z₃).

The point of reflection for this ray is somewhere on the surface of the mirror, say (x₂, y₂, 0). This distance traveled from the point 1 to the reflection point 2 is

\[ d_{12} = \sqrt{(0 - x_2)^2 + (0 - y_2)^2 + (z_1 - 0)^2} = \sqrt{x_2^2 + y_2^2 + z_1^2} \]

and the distance traveled from the reflection point 2 to the final point 3 is

\[ d_{23} = \sqrt{(x_2 - 0)^2 + (y_2 - y_3)^2 + (0 - z_3)^2} = \sqrt{x_2^2 + (y_2 - y_3)^2 + z_3^2} \]

The only point which is free to move is the reflection point, (x₂, y₂, 0), and that point can only move in the xy plane. Fermat’s principle states that the reflection point will be such to minimize the total distance,

\[ d_{12} + d_{23} = \sqrt{x_2^2 + y_2^2 + z_1^2} + \sqrt{x_2^2 + (y_2 - y_3)^2 + z_3^2} \]

We do this minimization by taking the partial derivative with respect to both x₂ and y₂. But we can do part by inspection alone. Any non-zero value of x₂ can only add to the total distance, regardless of the value of any of the other quantities. Consequently, x₂ = 0 is one of the conditions for minimization.

We are done! Although you are invited to finish the minimization process, once we know that x₂ = 0 we have that point 1, point 2, and point 3 all lie in the yz plane. The normal is parallel to the z axis, so it also lies in the yz plane. Everything is then in the yz plane.

E39-12 Refer to Page 442 of Volume 1.

E39-13 (a) \( \theta_1 = 38^\circ \).
(b) \( (1.58) \sin(38^\circ) = (1.22) \sin \theta_2 \). Then \( \theta_2 = \arcsin(0.797) = 52.9^\circ \).

E39-14 \( n_e = n_\nu \sin \theta_1 / \sin \theta_2 = (1.00) \sin(32.5^\circ) / \sin(21.0^\circ) = 1.50 \).

E39-15 \( n = c/v = (3.00 \times 10^8 \text{m/s})/(1.92 \times 10^8 \text{m/s}) = 1.56 \).

E39-16 \( v = c/n = (3.00 \times 10^8 \text{m/s})/(1.46) = 2.05 \times 10^8 \text{m/s} \).

E39-17 The speed of light in a substance with index of refraction \( n \) is given by \( v = c/n \). An electron will then emit Cerenkov radiation in this particular liquid if the speed exceeds

\[ v = c/n = (3.00 \times 10^8 \text{ m/s})/(1.54) = 1.95 \times 10^8 \text{ m/s} \].

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E39-18 Since \( t = \frac{d}{v} = \frac{n d}{c} \), \( \Delta t = \Delta n \frac{d}{c} \). Then
\[
\Delta t = \frac{(1.00029 - 1.00000)(1.61 \times 10^3 \text{m})}{(3.00 \times 10^8 \text{m/s})} = 1.56 \times 10^{-9} \text{s}.
\]

E39-19 The angle of the refracted ray is \( \theta_2 = 90^\circ \), the angle of the incident ray can be found by trigonometry,
\[
\tan \theta_1 = \frac{1.14 \text{ m}}{0.85 \text{ m}} = 1.34,
\]

or \( \theta_1 = 53.3^\circ \).

We can use these two angles, along with the index of refraction of air, to find that the index of refraction of the liquid from Eq. 39-4,
\[
n_1 = n_2 \frac{\sin \theta_2}{\sin \theta_1} = (1.00) \frac{\sin(90^\circ)}{\sin(53.3^\circ)} = 1.25.
\]

There are no units attached to this quantity.

E39-20 For an equilateral prism \( \phi = 60^\circ \). Then
\[
n = \frac{\sin(\psi + \phi)/2}{\sin(\phi/2)} = \frac{\sin[(37^\circ) + (60^\circ)]/2}{\sin(30^\circ)/2} = 1.5.
\]

E39-21

E39-22 \( t = \frac{d}{v} \); but \( L/d = \cos \theta_2 = \sqrt{1 - \sin^2 \theta_2} \) and \( v = \frac{c}{n} \). Combining,
\[
t = \frac{nL}{c \sqrt{1 - \sin^2 \theta_2}} = \frac{n^2 L}{c \sqrt{n^2 - \sin^2 \theta_1}} = \frac{(1.63)^2(0.547 \text{ m})}{(3 \times 10^8 \text{m/s}) \sqrt{(1.63^2) - \sin^2(24^\circ)}} \approx 3.07 \times 10^{-9} \text{s}.
\]

E39-23 The ray of light from the top of the smokestack to the life ring is \( \theta_1 \), where \( \tan \theta_1 = \frac{L}{h} \) with \( h \) the height and \( L \) the distance of the smokestack.

Snell's law gives \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \), so
\[
\theta_1 = \arcsin[(1.33) \sin(27^\circ)/(1.00)] = 37.1^\circ.
\]

Then \( L = h \tan \theta_1 = (98 \text{ m}) \tan(37.1^\circ) = 74 \text{ m} \).

E39-24 The length of the shadow on the surface of the water is
\[
x_1 = \frac{(0.64 \text{ m})}{\tan(55^\circ)} = 0.448 \text{ m}.
\]

The ray of light which forms the "end" of the shadow has an angle of incidence of \( 35^\circ \), so the ray travels into the water at an angle of
\[
\theta_2 = \arcsin \left( \frac{(1.00)}{(1.33) \sin(35^\circ)} \right) = 25.5^\circ.
\]

The ray travels an additional distance
\[
x_2 = \frac{(2.00 \text{ m} - 0.64 \text{ m})}{\tan(90^\circ - 25.5^\circ)} = 0.649 \text{ m}.
\]

The total length of the shadow is
\[
(0.448 \text{ m}) + (0.649 \text{ m}) = 1.10 \text{ m}.
\]

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We'll rely heavily on the figure for our arguments. Let $x$ be the distance between the points on the surface where the vertical ray crosses and the bent ray crosses.

In this exercise we will take advantage of the fact that, for small angles $\theta$, $\sin \theta \approx \tan \theta \approx \theta$. In this approximation Snell's law takes on the particularly simple form $n_1 \theta_1 = n_2 \theta_2$. The two angles here are conveniently found from the figure,

\[ \theta_1 \approx \tan \theta_1 = \frac{x}{d}, \]

and

\[ \theta_2 \approx \tan \theta_2 = \frac{x}{d_{app}}. \]

Inserting these two angles into the simplified Snell's law, as well as substituting $n_1 = n$ and $n_2 = 1.0$,

\[ n_1 \theta_1 = n_2 \theta_2, \]
\[ \frac{n x}{d} = \frac{x}{d_{app}}, \]
\[ d_{app} = \frac{d}{n}. \]

(a) You need to address the issue of total internal reflection to answer this question.

(b) Rearrange

\[ n = \frac{\sin[\psi + \phi]/2}{\sin[\phi/2]} \]

and $\theta = (\psi + \phi)/2$ to get

\[ \theta = \arcsin (n \sin[\phi/2]) = \arcsin ((1.60) \sin[(60^\circ)/2]) = 53.1^\circ. \]

Use the results of Ex. 39-35. The apparent thickness of the carbon tetrachloride layer, as viewed by an observer in the water, is

\[ d_{c,w} = n_w d_c / n_c = (1.33)(41 \text{ mm})/(1.46) = 37.5 \text{ mm}. \]

The total "thickness" from the water perspective is then $(37.5 \text{ mm}) + (20 \text{ mm}) = 57.5 \text{ mm}$. The apparent thickness of the entire system as view from the air is then

\[ d_{app} = (57.5 \text{ mm})/(1.33) = 43.2 \text{ mm}. \]
The frequency observed by the detector from the first source is (Eq. 39-31)

\[ f = f_1 \sqrt{1 - (0.717)^2} = 0.697 f_1. \]

The frequency observed by the detector from the second source is (Eq. 39-30)

\[ f = \frac{f_2 \sqrt{1 - (0.717)^2}}{1 + (0.717) \cos \theta} = \frac{0.697 f_2}{1 + (0.717) \cos \theta}. \]

We need to equate these and solve for \( \theta \). Then

\begin{align*}
0.697 f_1 &= \frac{0.697 f_2}{1 + 0.717 \cos \theta}, \\
1 + 0.717 \cos \theta &= f_2 / f_1, \\
\cos \theta &= (f_2 / f_1 - 1) / 0.717, \\
\theta &= 101.1^\circ.
\end{align*}

Subtract from 180° to find the angle with the line of sight.

P39-1 Consider the triangle in Fig. 39-45. The true position corresponds to the speed of light, the opposite side corresponds to the velocity of earth in the orbit. Then

\[ \theta = \arctan(29.8 \times 10^3 \text{m/s})/(3.00 \times 10^8 \text{m/s}) = 20.5^\circ. \]

P39-2 The distance to Jupiter from point \( x \) is \( d_x = r_x - r_E \). The distance to Jupiter from point \( y \) is

\[ d_y = \sqrt{r_x^2 + r_y^2}. \]

The difference in distance is related to the time according to

\[ (d_y - d_x)/t = c, \]

so

\[ \sqrt{(778 \times 10^8 \text{m})^2 + (150 \times 10^9 \text{m})^2} - (778 \times 10^8 \text{m}) + (150 \times 10^9 \text{m}) = 2.7 \times 10^8 \text{m/s}. \]

P39-3 \( \sin(30^\circ)/(4.0 \text{ m/s}) = \sin(\theta)/(3.0 \text{ m/s}) \). Then \( \theta = 22^\circ \). Water waves travel more slowly in shallower water, which means they always bend toward the normal as they approach land.

P39-4 (a) If the ray is normal to the water's surface then it passes into the water undeflected. Once in the water the problem is identical to Sample Problem 39-2. The reflected ray in the water is parallel to the incident ray in the water, so it also strikes the water normal, and is transmitted normal.

(b) Assume the ray strikes the water at an angle \( \theta_1 \). It then passes into the water at an angle \( \theta_2 \), where

\[ n_w \sin \theta_2 = n_a \sin \theta_1. \]

Once the ray is in the water then the problem is identical to Sample Problem 39-2. The reflected ray in the water is parallel to the incident ray in the water, so it also strikes the water at an angle \( \theta_2 \). When the ray travels back into the air it travels with an angle \( \theta_3 \), where

\[ n_w \sin \theta_2 = n_a \sin \theta_3. \]

Comparing the two equations yields \( \theta_1 = \theta_3 \), so the outgoing ray in the air is parallel to the incoming ray.
(a) As was done in Ex. 39-25 above we use the small angle approximation of
\[
\sin \theta \approx \theta \approx \tan \theta
\]
The incident angle is \( \theta \); if the light were to go in a straight line we would expect it to strike a
distance \( y_1 \) beneath the normal on the right hand side. The various distances are related to the
angle by
\[
\theta \approx \tan \theta \approx y_1/t.
\]
The light, however, does not go in a straight line, it is refracted according to (the small angle
approximation to) Snell’s law, \( n_1 \theta_1 = n_2 \theta_2 \), which we will simplify further by letting \( \theta_1 = \theta \), \( n_2 = n \),
and \( n_1 = 1 \), \( \theta = n \theta_2 \). The point where the refracted ray does strike is related to the angle by
\( \theta_2 \approx \tan \theta_2 = y_2/t \). Combining the three expressions,
\[
y_1 = ny_2.
\]
The difference, \( y_1 - y_2 \) is the vertical distance between the displaced ray and the original ray as
measured on the plate glass. A little algebra yields
\[
y_1 - y_2 = y_1 - y_1/n, \\
= y_1 (1 - 1/n), \\
= \frac{y_1}{n} - \frac{1}{n}.
\]
The perpendicular distance \( x \) is related to this difference by
\[
\cos \theta = x/(y_1 - y_2).
\]
In the small angle approximation \( \cos \theta \approx 1 - \theta^2/2 \). If \( \theta \) is sufficiently small we can ignore the square
term, and \( x \approx y_2 - y_1 \).

(b) Remember to use radians and not degrees whenever the small angle approximation is applied.
Then
\[
x = (1.0 \text{ cm})(0.175 \text{ rad}) \frac{(1.52) - 1}{(1.52)} = 0.060 \text{ cm}.
\]

P39-6  (a) At the top layer,
\[
n_1 \sin \theta_1 = \sin \theta;
\]
at the next layer,
\[
n_2 \sin \theta_2 = n_1 \sin \theta_1;
\]
at the next layer,
\[
n_3 \sin \theta_3 = n_2 \sin \theta_2.
\]
Combining all three expressions,
\[
n_3 \sin \theta_3 = \sin \theta.
\]
(b) \( \theta_3 = \arcsin[\sin(50°)/(1.0029)] = 49.98° \). Then shift is \( (50°) - (49.98°) = 0.02° \).

P39-7  The “big idea” of Problem 6 is that when light travels through layers the angle that it
makes in any layer depends only on the incident angle, the index of refraction where that incident
angle occurs, and the index of refraction at the current point.

That means that light which leaves the surface of the runway at 90° to the normal will make an
angle
\[
n_0 \sin 90° = n_0(1 + ay) \sin \theta
\]
at some height \( y \) above the runway. It is mildly entertaining to note that the value of \( n_0 \) is unimportant, only the value of \( a! \)

The expression

\[
\sin \theta = \frac{1}{1 + ay} \approx 1 - ay
\]

can be used to find the angle made by the curved path against the normal as a function of \( y \). The slope of the curve at any point is given by

\[
\frac{dy}{dx} = \tan(90^\circ - \theta) = \cot \theta = \frac{\cos \theta}{\sin \theta}.
\]

Now we need to know \( \cos \theta \). It is

\[
\cos \theta = \sqrt{1 - \sin^2 \theta} \approx \sqrt{2ay}.
\]

Combining

\[
\frac{dy}{dx} \approx \frac{\sqrt{2ay}}{1 - ay},
\]

and now we integrate. We will ignore the \( ay \) term in the denominator because it will always be small compared to 1. Then

\[
\int_0^d dx = \int_0^h \frac{dy}{\sqrt{2ay}}.
\]

\[
d = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2(1.7 \text{ m})}{(1.5 \times 10^{-6} \text{ m}^{-1})}} = 1500 \text{ m}.
\]

P39-8 The energy of a particle is given by \( E^2 = p^2 c^2 + m^2 c^4 \). This energy is related to the mass by \( E = \gamma mc^2 \). \( \gamma \) is related to the speed by \( \gamma = 1/\sqrt{1 - u^2/c^2} \). Rearranging,

\[
\frac{u}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{m^2c^2}{p^2 + m^2c^2}}.
\]

Since \( n = c/u \) we can write this as

\[
n = \sqrt{1 + \frac{m^2c^2}{p^2}} = \sqrt{1 + \left(\frac{mc^2}{pc}\right)^2}.
\]

For the pion,

\[
n = \sqrt{1 + \left(\frac{135 \text{ MeV}}{145 \text{ MeV}}\right)^2} = 1.37.
\]

For the muon,

\[
n = \sqrt{1 + \left(\frac{106 \text{ MeV}}{145 \text{ MeV}}\right)^2} = 1.24.
\]
P39-9  (a) Before adding the drop of liquid project the light ray along the angle \( \theta \) so that \( \theta = 0 \). Increase \( \theta \) slowly until total internal reflection occurs at angle \( \theta_1 \). Then

\[ n_g \sin \theta_1 = 1 \]

is the equation which can be solved to find \( n_g \).

Now put the liquid on the glass and repeat the above process until total internal reflection occurs at angle \( \theta_2 \). Then

\[ n_g \sin \theta_2 = n_l. \]

Note that \( n_g < n_l \) for this method to work.

(b) This is not terribly practical.

P39-10  Let the internal angle at \( Q \) be \( \theta_Q \). Then \( n \sin \theta_Q = 1 \), because it is a critical angle. Let the internal angle at \( P \) be \( \theta_P \). Then \( \theta_P + \theta_Q = 90^\circ \). Combine this with the other formula and

\[ 1 = n \sin(90 - \theta_P) = n \cos \theta_Q = n \sqrt{1 - \sin^2 \theta_P}. \]

Not only that, but \( \sin \theta_1 = n \sin \theta_P \), or

\[ 1 = n \sqrt{1 - (\sin \theta_1)^2/n^2}, \]

which can be solved for \( n \) to yield

\[ n = \sqrt{1 + \sin^2 \theta_1}. \]

(b) The largest value of the sine function is one, so \( n_{\text{max}} = \sqrt{2} \).

P39-11  (a) The fraction of light energy which escapes from the water is dependent on the critical angle. Light radiates in all directions from the source, but only that which strikes the surface at an angle less than the critical angle will escape. This critical angle is

\[ \sin \theta_c = 1/n. \]

We want to find the solid angle of the light which escapes; this is found by integrating

\[ \Omega = \int_0^{2\pi} \int_0^{\theta_c} \sin \theta \, d\theta \, d\phi. \]

This is not a hard integral to do. The result is

\[ \Omega = 2\pi(1 - \cos \theta_c). \]

There are \( 4\pi \) steradians in a spherical surface, so the fraction which escapes is

\[ f = \frac{1}{2}(1 - \cos \theta_c) = \frac{1}{2}(1 - \sqrt{1 - \sin^2 \theta_c}). \]

The last substitution is easy enough. We never needed to know the depth \( h \).

(b) \( f = \frac{1}{2}(1 - \sqrt{1 - (1/(1.3))^2}) = 0.18 \).
In this problem we look for the location of the third-order bright fringe, so
\[ \theta = \sin^{-1} \frac{m\lambda}{d} = \sin^{-1} \frac{(3)(554 \times 10^{-9} \text{m})}{(7.7 \times 10^{-6} \text{m})} = 12.5^\circ = 0.22 \text{ rad.} \]

\[ d_1 \sin \theta = \lambda \] gives the first maximum; \( d_2 \sin \theta = 2\lambda \) puts the second maximum at the location of the first. Divide the second expression by the first and \( d_2 = 2d_1 \). This is a 100\% increase in \( d \).

\[ \Delta y = \lambda D/d = \frac{(512 \times 10^{-9} \text{m})(5.4 \text{ m})}{(1.2 \times 10^{-3} \text{m})} = 2.3 \times 10^{-3} \text{m}. \]

\[ d = \lambda'/\sin \theta = \frac{(592 \times 10^{-9} \text{m})}{\sin(1.00^\circ)} = 3.39 \times 10^{-5} \text{m}. \]

Since the angles are very small, we can assume \( \sin \theta \approx \theta \) for angles measured in radians.

If the interference fringes are 0.23° apart, then the angular position of the first bright fringe is 0.23° away from the central maximum. Eq. 41-1, written with the small angle approximation in mind, is \( d\theta = \lambda \) for this first \((m = 1)\) bright fringe. The goal is to find the wavelength which increases \( \theta \) by 10%. To do this we must increase the right hand side of the equation by 10%, which means increasing \( \lambda \) by 10%. The new wavelength will be \( \lambda' = 1.1\lambda = 1.1(589 \text{ nm}) = 650 \text{ nm} \).

Immersing the apparatus in water will shorten the wavelengths to \( \lambda/n \). Start with \( d\sin \theta_0 = \lambda \); and then find \( \theta \) from \( d\sin \theta = \lambda/n \). Combining the two expressions,
\[ \theta = \arcsin[\sin \theta_0/n] = \arcsin[\sin(0.20^\circ)/(1.33)] = 0.15^\circ. \]

The third-order fringe for a wavelength \( \lambda \) will be located at \( y = 3\lambda D/d \), where \( y \) is measured from the central maximum. Then \( \Delta y \) is
\[ y_1 - y_2 = 3(\lambda_1 - \lambda_2)D/d = 3(612 \times 10^{-9} \text{m} - 480 \times 10^{-9} \text{m})(1.36 \text{ m})/(5.22 \times 10^{-3} \text{m}) = 1.03 \times 10^{-4} \text{m}, \]

\[ \theta = \arctan(y/D); \]
\[ \lambda = d\sin \theta = (0.120 \text{ m})\sin[\arctan(0.180 \text{ m}/2.0 \text{ m})] = 1.08 \times 10^{-2} \text{m}. \]

Then \( f = v/\lambda = (0.25 \text{ m/s})/(1.08 \times 10^{-2} \text{m}) = 23 \text{ Hz}. \)

A variation of Eq. 41-3 is in order:
\[ y_m = \left( m + \frac{1}{2} \right) \frac{\lambda D}{d} \]

We are given the distance (on the screen) between the first minima \((m = 0)\) and the tenth minima \((m = 9)\). Then
\[ 18 \text{ mm} = y_9 - y_0 = 9 \frac{\lambda(50 \text{ cm})}{(0.15 \text{ mm})}, \]
or \( \lambda = 6 \times 10^{-4} \text{ mm} = 600 \text{ nm} \).

The "maximum" maxima is given by the integer part of
\[ m = d\sin(90^\circ)/\lambda = (2.0 \text{ m})/(0.50 \text{ m}) = 4. \]

Since there is no integer part, the "maximum" maxima occurs at 90°. These are point sources radiating in both directions, so there are two central maxima, and four maxima each with \( m = 1 \), \( m = 2 \), and \( m = 3 \). But the \( m = 4 \) values overlap at 90°, so there are only two. The total is 16.
\[ \Delta y = \frac{\lambda D}{d} = \frac{(589 \times 10^{-9} \text{m})(1.13 \text{ m})}{(0.18 \times 10^{-3} \text{m})} = 3.70 \times 10^{-3} \text{m}. \]

\( E41-12 \) Consider Fig. 41-5, and solve it **exactly** for the information given. For the tenth bright fringe \( r_1 = 10\lambda + r_2 \). There are two important triangles:

\[ r_2^2 = D^2 + (y - d/2)^2 \]

and

\[ r_1^2 = D^2 + (y + d/2)^2 \]

Solving to eliminate \( r_2 \),

\[ \sqrt{D^2 + (y + d/2)^2} = \sqrt{D^2 + (y - d/2)^2} + 10\lambda. \]

This has solution

\[ y = 5\lambda \sqrt{\frac{4D^2 + d^2 - 100\lambda^2}{d^2 - 100\lambda^2}}. \]

The solution predicted by Eq. 41-1 is

\[ y' = \frac{10\lambda}{d} \sqrt{D^2 + y'^2}, \]

or

\[ y' = 5\lambda \sqrt{\frac{4D^2}{d^2 - 100\lambda^2}}. \]

The fractional error is \( y'/y - 1 \), or

\[ \sqrt{\frac{4D^2}{4D^2 + d^2 - 100\lambda^2}} - 1, \]

or

\[ \sqrt{\frac{4(40 \text{ mm})^2}{4(40 \text{ mm})^2 + (2 \text{ mm})^2 - 100(589 \times 10^{-6} \text{mm})^2}} - 1 = -3.1 \times 10^{-4}. \]

\( E41-14 \) (a) \( \Delta x = c/\Delta t = (3.00 \times 10^8 \text{m/s})/(1 \times 10^{-8} \text{s}) = 3 \text{ m}. \)

(b) No.
E41-34 \[(1.42\text{ cm}) = \sqrt{(10 - \frac{1}{2})R\lambda}, \text{ while } (1.27\text{ cm}) = \sqrt{(10 - \frac{1}{2})R\lambda/n}. \text{ Divide one expression by the other, and } (1.42\text{ cm})/(1.27\text{ cm}) = \sqrt{n}, \text{ or } n = 1.25.\]

E41-35 \[(0.162\text{ cm}) = \sqrt{n - \frac{1}{2}}R\lambda, \text{ while } (0.368\text{ cm}) = \sqrt{n + 20 - \frac{1}{2}}R\lambda. \text{ Square both expressions, the divide one by the other, and find}\]
\[
\frac{(n + 19.5)}{(n - 0.5)} = \frac{(0.368\text{ cm}/0.162\text{ cm})^2}{5.16} = 5.16
\]

which can be rearranged to yield
\[
n = \frac{19.5 + 5.16 \times 0.5}{5.16 - 1} = 5.308.\]

Oops! That should be an integer, shouldn’t it? The above work is correct, which means that there really aren’t bright bands at the specified locations. I’m just going to gloss over that fact and solve for \(R\) using the value of \(m = 5.308\). Then
\[
R = \frac{r^2}{(m - 1/2)\lambda} = \frac{(0.162\text{ cm})^2}{(5.308 - 0.5)(546\text{ nm})} = 1.00\text{ m}.\]

Well, at least we got the answer which is in the back of the book...

E41-36 Pretend the ship is a two point source emitter, one \(h\) above the water, and one \(h\) below the water. The one below the water is out of phase by half a wavelength. Then \(d\sin \theta = \lambda\), where \(d = 2h\), gives the angle for theta for the first minimum.
\[
\frac{\lambda}{2h} = \frac{(3.43\text{ m})}{(23\text{ m})} = 7.46 \times 10^{-2} = \sin \theta \approx H/D,
\]
so \(D = (160\text{ m})/(7.46 \times 10^{-2}) = 2.14\text{ km}\).

E41-37 The phase difference is \(2\pi/\lambda_n\) times the path difference which is \(2d\), so
\[
\phi = 4\pi d/\lambda_n = 4\pi nd/\lambda.
\]

We are given that \(d = 100 \times 10^{-9}\text{ m}\) and \(n = 1.38\).
(a) \(\phi = 4\pi(1.38)(100 \times 10^{-9}\text{ m})/((450 \times 10^{-9}\text{ m})) = 3.85\). Then
\[
\frac{I}{I_0} = \cos^2 \left(\frac{3.85}{2}\right) = 0.12.
\]

The reflected ray is diminished by \(1 - 0.12 = 88\%\).
(b) \(\phi = 4\pi(1.38)(100 \times 10^{-9}\text{ m})/((650 \times 10^{-9}\text{ m})) = 2.67\). Then
\[
\frac{I}{I_0} = \cos^2 \left(\frac{2.67}{2}\right) = 0.055.
\]

The reflected ray is diminished by \(1 - 0.055 = 95\%\).

E41-38 The change in the optical path length is \(2(d - d/n)\), so \(7\lambda/n = 2d(1 - 1/n), \text{ or}\)
\[
d = \frac{7(589 \times 10^{-9}\text{ m})}{2(1.42) - 2} = 4.9 \times 10^{-6}\text{ m}.
\]
E41-39 When $M_2$ moves through a distance of $\lambda/2$ a fringe has will be produced, destroyed, and then produced again. This is because the light travels twice through any change in distance. The wavelength of light is then

$$\lambda = \frac{2(0.233 \text{ mm})}{792} = 588 \text{ nm}.$$ 

E41-40 The change in the optical path length is $2(d - d/n)$, so $60\lambda = 2d(1 - 1/n)$, or

$$n = \frac{1}{1 - 60\lambda/2d} = \frac{1}{1 - 60(500 \times 10^{-9}\text{m})/2(5 \times 10^{-2}\text{m})} = 1.00030.$$ 

P41-1 (a) This is a small angle problem, so we use Eq. 41-4. The distance to the screen is $2 \times 20\text{ m}$, because the light travels to the mirror and back again. Then

$$d = \frac{\Delta d}{\Delta y} = \frac{(632.8 \text{ nm})(40.0\text{ m})}{(0.1\text{ m})} = 0.253\text{ mm}.$$ 

(b) Placing the cellophane over one slit will cause the interference pattern to shift to the left or right, but not disappear or change size. How does it shift? Since we are picking up 2.5 waves then we are, in effect, swapping bright fringes for dark fringes.

P41-2 The change in the optical path length is $d - d/n$, so $7\lambda/n = d(1 - 1/n)$, or

$$d = \frac{7(550 \times 10^{-9}\text{m})}{(1.58) - 1} = 6.64 \times 10^{-6}\text{m}.$$ 

P41-3 The distance from $S_1$ to $P$ is $r_1 = \sqrt{(x + d/2)^2 + y^2}$. The distance from $S_2$ to $P$ is $r_2 = \sqrt{(x - d/2)^2 + y^2}$. The difference in distances is fixed at some value, say $c$, so that

$$r_1 - r_2 = c,$$

$$r_1^2 - 2r_1r_2 + r_2^2 = c^2,$$

$$(r_1^2 + r_2^2 - c^2)^2 = 4r_1^2r_2^2,$$

$$(r_1^2 + r_2^2 - c^2)^2 - 2c^2(r_1^2 + r_2^2) + c^4 = 0,$$

$$2c^2r_1^2 + 2c^2r_2^2 + d^2/2 + 2xy + c^4 = 0,$$

$$4x^2d^2 - 4c^2x^2 - c^2d^2 - 4c^2y^2 + c^4 = 0,$$

$$4(c^2 - c^2)x^2 - 4c^2y^2 = c^2(d^2 - c^2).$$

Yes, that is the equation of a hyperbola.

P41-4 The change in the optical path length for each slit is $nt - t$, where $n$ is the corresponding index of refraction. The net change in the path difference is then $n_2t - n_1t$. Consequently, $m\lambda = t(n_2 - n_1)$, so

$$t = \frac{(5)(480 \times 10^{-9}\text{m})}{(1.7) - (1.4)} = 8.0 \times 10^{-6}\text{m}.$$ 

P41-5 The intensity is given by Eq. 41-17, which, in the small angle approximation, can be written as

$$I_\theta = 4I_0 \cos^2 \left( \frac{\pi d\theta}{\lambda} \right).$$

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The intensity will be half of the maximum when
\[
\frac{1}{2} = \cos^2 \left( \frac{\pi d \Delta \theta / 2}{\lambda} \right)
\]
or
\[
\frac{\pi}{4} = \frac{\pi d \Delta \theta}{2\lambda},
\]
which will happen if \(\Delta \theta = \lambda/2d\).

**P41-6** Follow the construction in Fig. 41-10, except that one of the electric field amplitudes is twice the other. The resultant field will have a length given by
\[
E' = \sqrt{(2E_0 + E_0 \cos \phi)^2 + (E_0 \sin \phi)^2},
\]
\[
= E_0 \sqrt{5 + 4 \cos \phi},
\]
so squaring this yields
\[
I = I_0 \left( 5 + 4 \cos 2 \frac{\pi d \sin \theta}{\lambda} \right),
\]
\[
= I_0 \left( 1 + 8 \cos^2 \frac{\pi d \sin \theta}{\lambda} \right),
\]
\[
= \frac{I_m}{9} \left( 1 + 8 \cos^2 \frac{\pi d \sin \theta}{\lambda} \right).
\]

**P41-7** We actually did this problem in Exercise 41-27, although slightly differently. One maximum is
\[
2(1.32)d = (m - 1/2)(679 \text{ nm}),
\]
the other is
\[
2(1.32)d = (m + 1/2)(485 \text{ nm}).
\]
Set these equations equal to each other,
\[
(m - 1/2)(679 \text{ nm}) = (m + 1/2)(485 \text{ nm}),
\]
and find \(m = 3\). Then the thickness is
\[
d = (3 - 1/2)(679 \text{ nm})/2(1.32) = 643 \text{ nm}.
\]

**P41-8** (a) Since we are concerned with transmission there is a phase shift for two rays, so
\[
2d = m\lambda_n
\]
The minimum thickness occurs when \(m = 1\); solving for \(d\) yields
\[
d = \frac{\lambda}{2n} = \frac{(525 \times 10^{-9}\text{m})}{2(1.55)} = 169 \times 10^{-9}\text{m}.
\]
(b) The wavelengths are different, so the other parts have differing phase differences.
(c) The nearest destructive interference wavelength occurs when \(m = 1.5\), or
\[
\lambda = 2nd = 2(1.55)(1.5)(169 \times 10^{-9}\text{m}) = 393 \times 10^{-9}\text{m}.
\]
This is blue-violet.
P41-9 It doesn't matter if we are looking at bright are dark bands. It doesn't even matter if we concern ourselves with phase shifts. All that cancels out. Consider $2\delta d = \delta m \lambda$; then

$$\delta d = (10)(480 \text{ nm})/2 = 2.4 \mu\text{m}.$$ 

P41-10 (a) Apply $2d = m\lambda$. Then

$$d = (7)(600 \times 10^{-9} \text{ m})/2 = 2100 \times 10^{-9} \text{ m}.$$ 

(b) When water seeps in it introduces an extra phase shift. Point A becomes then a bright fringe, and the equation for the number of bright fringes is $2nd = m\lambda$. Solving for $m$,

$$m = 2(1.33)(2100 \times 10^{-9} \text{ m})/(600 \times 10^{-6} \text{ m}) = 9.3;$$

this means that point B is almost, but not quite, a dark fringe, and there are nine of them.

P41-11 (a) Look back at the work for Sample Problem 41-5 where it was found

$$r_m = \sqrt{(m - \frac{1}{2})\lambda R},$$

We can write this as

$$r_m = \sqrt{(1 - \frac{1}{2m}) m\lambda R}$$

and expand the part in parentheses in a binomial expansion,

$$r_m \approx \left(1 - \frac{1}{2m}\right)^{\frac{1}{2}} \sqrt{m\lambda R}.$$ 

We will do the same with

$$r_{m+1} = \sqrt{(m + 1 - \frac{1}{2})\lambda R},$$

expanding

$$r_{m+1} = \sqrt{(1 + \frac{1}{2m}) m\lambda R}$$

to get

$$r_{m+1} \approx \left(1 + \frac{1}{2m}\right)^{\frac{1}{2}} \sqrt{m\lambda R}.$$ 

Then

$$\Delta r \approx \frac{1}{2m} \sqrt{m\lambda R},$$

or

$$\Delta r \approx \frac{1}{2} \sqrt{\lambda R/m}.$$ 

(b) The area between adjacent rings is found from the difference,

$$A = \pi \left(r_{m+1}^2 - r_m^2\right),$$

and into this expression we will substitute the exact values for $r_m$ and $r_{m+1}$,

$$A = \pi \left((m + 1 - \frac{1}{2})\lambda R - (m - \frac{1}{2})\lambda R\right),$$

$$= \pi \lambda R.$$

Unlike part (a), we did not need to assume $m \gg 1$ in order to arrive at this expression; it is exact for all $m$. 

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The path length shift that occurs when moving the mirror at distance $x$ is $2x$. This means $\phi = \frac{2\pi 2x}{\lambda} = \frac{4\pi x}{\lambda}$. The intensity is then

$$I = 4I_0 \cos^2 \frac{2\pi x}{\lambda}.$$
E42-1 \( \lambda = a \sin \theta = (0.022 \text{ mm}) \sin(1.8^\circ) = 6.91 \times 10^{-7} \text{ m}. \)

E42-2 \( a = \lambda / \sin \theta = (0.10 \times 10^{-9} \text{ m}) / \sin(0.12 \times 10^{-3} \text{ rad})/2 = 1.7 \mu \text{ m}. \)

E42-3 (a) This is a valid small angle approximation problem: the distance between the points on the screen is much less than the distance to the screen. Then
\[
\theta \approx \frac{(0.0162 \text{ m})}{(2.16 \text{ m})} = 7.5 \times 10^{-3} \text{ rad.}
\]

(b) The diffraction minima are described by Eq. 42-3,
\[
a \sin \theta = m\lambda,
\]
\[
a \sin(7.5 \times 10^{-3} \text{ rad}) = (2)(441 \times 10^{-9} \text{ m}),
\]
\[
a = 1.18 \times 10^{-4} \text{ m.}
\]

E42-4 \( a = \lambda / \sin \theta = (633 \times 10^{-9} \text{ m}) / \sin(1.97^\circ/2) = 36.8 \mu \text{ m}. \)

E42-5 (a) We again use Eq. 42-3, but we will need to throw in a few extra subscripts to distinguish between which wavelength we are dealing with. If the angles match, then so will the sine of the angles. We then have \( \sin \theta_{a,1} = \sin \theta_{b,2} \) or, using Eq. 42-3,
\[
\frac{(1)\lambda_a}{a} = \frac{(2)\lambda_b}{a},
\]
from which we can deduce \( \lambda_a = 2\lambda_b. \)

(b) Will any other minima coincide? We want to solve for the values of \( m_a \) and \( m_b \) that will be integers and have the same angle. Using Eq. 42-3 one more time,
\[
\frac{m_a\lambda_a}{a} = \frac{m_b\lambda_b}{a},
\]
and then substituting into this the relationship between the wavelengths, \( m_a = m_b/2 \). whenever \( m_b \) is an even integer \( m_a \) is an integer. Then all of the diffraction minima from \( \lambda_a \) are overlapped by a minima from \( \lambda_b \).

E42-6 The angle is given by \( \sin \theta = 2\lambda/a. \) This is a small angle, so we can use the small angle approximation of \( \sin \theta = y/D. \) Then
\[
y = 2D\lambda/a = 2(0.714 \text{ m})(593 \times 10^{-9} \text{ m})/(420 \times 10^{-6} \text{ m}) = 2.02 \text{ mm.}
\]

E42-7 Small angles, so \( y/D = \sin \theta = \lambda/a. \) Then
\[
a = D\lambda/y = (0.823 \text{ m})(546 \times 10^{-9} \text{ m})/(5.20 \times 10^{-3} \text{ m}/2) = 1.73 \times 10^{-4} \text{ m.}
\]

E42-8 (b) Small angles, so \( \Delta y/D = \Delta m\lambda/a. \) Then
\[
a = \Delta mD\lambda/\Delta y = (5 - 1)(0.413 \text{ m})(546 \times 10^{-9} \text{ m})/(0.350 \times 10^{-3} \text{ m}) = 2.58 \text{ mm.}
\]

(a) \( \theta = \arcsin(\lambda/a) = \arcsin[(546 \times 10^{-9} \text{ m})/(2.58 \text{ mm})] = 1.21 \times 10^{-2^\circ}. \)

E42-9 Small angles, so \( \Delta y/D = \Delta m\lambda/a. \) Then
\[
\Delta y = \Delta mD\lambda/a = (2 - 1)(2.94 \text{ m})(589 \times 10^{-9} \text{ m})/(1.16 \times 10^{-3} \text{ m}) = 1.49 \times 10^{-3} \text{ m.}
\]
E42-10 Doubling the width of the slit results in a narrowing of the diffraction pattern. Since the width of the central maximum is effectively cut in half, then there is twice the energy in half the space, producing four times the intensity.

E42-11 (a) This is a small angle approximation problem, so

\[ \theta = \frac{(1.13 \text{ cm})}{(3.48 \text{ m})} = 3.25 \times 10^{-3} \text{ rad}. \]

(b) A convenient measure of the phase difference, \( \alpha \), is related to \( \theta \) through Eq. 42-7,

\[ \alpha = \frac{\pi a}{\lambda} \sin \theta = \frac{\pi (25.2 \times 10^{-6} \text{ m})}{(538 \times 10^{-9} \text{ m})} \sin (3.25 \times 10^{-3} \text{ rad}) = 0.478 \text{ rad} \]

(c) The intensity at a point is related to the intensity at the central maximum by Eq. 42-8,

\[ \frac{I_\theta}{I_m} = \left( \frac{\sin \alpha}{\alpha} \right)^2 = \left( \frac{\sin (0.478 \text{ rad})}{0.478 \text{ rad}} \right)^2 = \frac{926}{1} \]

E42-12 Consider Fig. 42-11, the angle with the vertical is given by \( (\pi - \phi)/2 \). For Fig. 42-10(d) the circle has wrapped once around onto itself so the angle with the vertical is \( (3\pi - \phi)/2 \). Substitute \( \alpha \) into this expression and the angle against the vertical is \( 3\pi/2 - \alpha \).

Use the result from Problem 42-3 that \( \tan \alpha = \alpha \) for the maxima. The lowest non-zero solution is \( \alpha = 4.49341 \text{ rad} \). The angle against the vertical is then 0.21898 rad, or 12.5°.

E42-13 Drawing heavily from Sample Problem 42-4,

\[ \theta_x = \arcsin \left( \frac{\alpha \pi \lambda}{\pi a} \right) = \arcsin \left( \frac{1.39}{10\pi} \right) = 2.54^\circ. \]

Finally, \( \Delta \theta = 2\theta_x = 5.1^\circ \).

E42-14 (a) Rayleigh's criterion for resolving images (Eq. 42-11) requires that two objects have an angular separation of at least

\[ \theta_R = \sin^{-1} \left( \frac{1.22\lambda}{d} \right) = \sin^{-1} \left( \frac{1.22(540 \times 10^{-9})}{(4.90 \times 10^{-3} \text{ m})} \right) = 1.34 \times 10^{-4} \text{ rad} \]

(b) The linear separation is \( y = \theta D = (1.34 \times 10^{-4} \text{ rad})(163 \times 10^3 \text{ m}) = 21.9 \text{ m} \).

E42-15 (a) Rayleigh's criterion for resolving images (Eq. 42-11) requires that two objects have an angular separation of at least

\[ \theta_R = \sin^{-1} \left( \frac{1.22\lambda}{d} \right) = \sin^{-1} \left( \frac{1.22(562 \times 10^{-9})}{(5.00 \times 10^{-3} \text{ m})} \right) = 1.37 \times 10^{-4} \text{ rad}. \]

(b) Once again, this is a small angle, so we can use the small angle approximation to find the distance to the car. In that case \( \theta_R = y/D \), where \( y \) is the headlight separation and \( D \) the distance to the car. Solving,

\[ D = y/\theta_R = (1.42 \text{ m})/(1.37 \times 10^{-4} \text{ rad}) = 1.04 \times 10^4 \text{ m}, \]

or about six or seven miles.
$y/D = 1.22\lambda/a$; or

\[ D = (5.20 \times 10^{-3}\text{m})(4.60 \times 10^{-3}/\text{m})/1.22(542 \times 10^{-9}\text{m}) = 36.2\text{ m}. \]

**E42-17** The smallest resolvable angular separation will be given by Eq. 42-11,

\[ \theta_R = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(565 \times 10^{-9}\text{m})}{(5.08\text{ m})}\right) = 1.36 \times 10^{-7}\text{ rad}, \]

The smallest objects resolvable on the Moon's surface by this telescope have a size $y$ where

\[ y = D\theta_R = (3.84 \times 10^6\text{ m})(1.36 \times 10^{-7}\text{ rad}) = 52.2\text{ m} \]

**E42-18** $y/D = 1.22\lambda/a$; or

\[ y = 1.22(1.57 \times 10^{-2}\text{m})(6.25 \times 10^3\text{m})/(2.33\text{ m}) = 51.4\text{ m} \]

**E42-19** $y/D = 1.22\lambda/a$; or

\[ D = (4.8 \times 10^{-2}\text{m})(4.3 \times 10^{-3}/\text{m})/1.22(0.12 \times 10^{-9}\text{m}) = 1.4 \times 10^6\text{ m} \]

**E42-20** $y/D = 1.22\lambda/a$; or

\[ d = 1.22(550 \times 10^{-9}\text{m})(160 \times 10^3\text{m})/(0.30\text{ m}) = 0.36\text{ m}. \]

**E42-21** Using Eq. 42-11, we find the minimum resolvable angular separation is given by

\[ \theta_R = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(475 \times 10^{-9}\text{m})}{(4.4 \times 10^{-3}\text{m})}\right) = 1.32 \times 10^{-4}\text{ rad} \]

The dots are 2 mm apart, so we want to stand a distance $D$ away such that

\[ D > y/\theta_R = (2 \times 10^{-3}\text{ m})/(1.32 \times 10^{-4}\text{ rad}) = 15\text{ m}. \]

**E42-22** $y/D = 1.22\lambda/a$; or

\[ y = 1.22(500 \times 10^{-9}\text{m})(354 \times 10^3\text{m})/(9.14\text{ m}/2) = 4.73 \times 10^{-2}\text{ m}. \]

**E42-23** (a) $\lambda = v/f$. Now use Eq. 42-11:

\[ \theta = \arcsin\left(\frac{1.22}{(1450\text{ m/s})/(25 \times 10^3\text{Hz})(0.60\text{ m})}\right) = 6.77^\circ. \]

(b) Following the same approach,

\[ \theta = \arcsin\left(\frac{1.22}{(1450\text{ m/s})/(1 \times 10^3\text{Hz})(0.60\text{ m})}\right) \]

has no real solution, so there is no minimum.
P42-4 The outgoing beam strikes the moon with a circular spot of radius

$$r = 1.22\lambda D/a = 1.22(0.69 \times 10^{-6}\text{m})(3.82 \times 10^8\text{m})/(2 \times 1.3\text{m}) = 123\text{m}.$$  

The light is not evenly distributed over this circle.

If $P_0$ is the power in the light, then

$$P_0 = \int I_0 r \, dr \, d\phi = 2\pi \int_0^R I_0 r \, dr,$$

where $R$ is the radius of the central peak and $I_0$ is the angular intensity. For $a \gg \lambda$ we can write $\alpha \approx \pi a r / \lambda D$, then

$$P_0 = 2\pi I_m \left(\frac{\lambda D}{\pi a}\right)^2 \int_0^{\pi/2} \frac{\sin^2 \alpha}{a} \, d\alpha \approx 2\pi I_m \left(\frac{\lambda D}{\pi a}\right)^2.$$(0.82)

Then the intensity at the center falls off with distance $D$ as

$$I_m = 1.9 \left(\frac{a}{\lambda D}\right)^2 P_0.$$

The fraction of light collected by the mirror on the moon is then

$$P_1/P_0 = 1.9 \left(\frac{(2 \times 1.3\text{m})}{(0.69 \times 10^{-6}\text{m})(3.82 \times 10^8\text{m})}\right)^2 \pi(0.10\text{m})^2 = 5.6 \times 10^{-6}.$$

The fraction of light collected by the mirror on the Earth is then

$$P_2/P_1 = 1.9 \left(\frac{(2 \times 0.10\text{m})}{(0.69 \times 10^{-6}\text{m})(3.82 \times 10^8\text{m})}\right)^2 \pi(1.3\text{m})^2 = 5.6 \times 10^{-6}.$$

Finally, $P_2/P_0 = 3 \times 10^{-11}$.

P42-5 (a) The ring is reddish because it occurs at the blue minimum.

(b) Apply Eq. 42-11 for blue light:

$$d = 1.22\lambda / \sin \theta = 1.22(400\text{ nm}) / \sin(0.375^\circ) = 70\text{ \mu m}.$$

(c) Apply Eq. 42-11 for red light:

$$\theta = \arcsin \left(1.22(700\text{ nm})/(70\text{ \mu m})\right) \approx 0.7^\circ,$$

which occurs 3 lunar radii from the moon.

P42-6 The diffraction pattern is a property of the speaker, not the interference between the speakers. The diffraction pattern should be unaffected by the phase shift. The interference pattern, however, should shift up or down as the phase of the second speaker is varied.

P42-7 (a) The missing fringe at $\theta = 5^\circ$ is a good hint as to what is going on. There should be some sort of interference fringe, unless the diffraction pattern has a minimum at that point. This would be the first minimum, so

$$a \sin(5^\circ) = (440 \times 10^{-6}\text{m})$$

would be a good measure of the width of each slit. Then $a = 5.05 \times 10^{-6}\text{m}$. 

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(b) If the diffraction pattern envelope were not present we could expect that the fourth interference maxima beyond the central maximum would occur at this point, and then

\[ d \sin(5^\circ) = 4(440 \times 10^{-9} \text{m}) \]

yielding

\[ d = 2.02 \times 10^{-5} \text{m}. \]

(c) Apply Eq. 42-17, where \( \beta = m\pi \) and,

\[ \alpha = \frac{\pi a}{\lambda} \sin \theta = \frac{\pi a m\lambda}{\lambda d} = m \frac{\pi a}{d} = m\pi/4. \]

Then for \( m = 1 \) we have

\[ I_1 = (7) \left( \frac{\sin(\pi/4)}{(\pi/4)} \right)^2 = 5.7; \]

while for \( m = 2 \) we have

\[ I_2 = (7) \left( \frac{\sin(2\pi/4)}{(2\pi/4)} \right)^2 = 2.8. \]

These are in good agreement with the figure.
E43-1  (a) \( d = \frac{21.5 \times 10^{-3} \text{m}}{6140} = 3.50 \times 10^{-6} \text{m} \).

(b) There are a number of angles allowed:

\[
\theta = \arcsin\left(\frac{1}{1}(589 \times 10^{-6} \text{m})/(3.50 \times 10^{-6} \text{m})\right) = 9.7^\circ,
\]

\[
\theta = \arcsin\left(\frac{2}{1}(589 \times 10^{-6} \text{m})/(3.50 \times 10^{-6} \text{m})\right) = 19.5^\circ,
\]

\[
\theta = \arcsin\left(\frac{3}{1}(589 \times 10^{-6} \text{m})/(3.50 \times 10^{-6} \text{m})\right) = 30.3^\circ,
\]

\[
\theta = \arcsin\left(\frac{4}{1}(589 \times 10^{-6} \text{m})/(3.50 \times 10^{-6} \text{m})\right) = 42.3^\circ,
\]

\[
\theta = \arcsin\left(\frac{5}{1}(589 \times 10^{-6} \text{m})/(3.50 \times 10^{-6} \text{m})\right) = 57.3^\circ.
\]

E43-2  The distance between adjacent rulings is

\[
d = \frac{(2)(612 \times 10^{-6} \text{m})}{\sin(33.2^\circ)} = 2.235 \times 10^{-6} \text{m}.
\]

The number of lines is then

\[
N = \frac{D}{d} = \frac{(2.86 \times 10^{-2} \text{m})}{(2.235 \times 10^{-6} \text{m})} = 12,800.
\]

E43-3  We want to find a relationship between the angle and the order number which is linear.

We'll plot the data in this representation, and then use a least squares fit to find the wavelength.

The data to be plotted is

<table>
<thead>
<tr>
<th>( m )</th>
<th>( \theta )</th>
<th>( \sin \theta )</th>
<th>( m )</th>
<th>( \theta )</th>
<th>( \sin \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.6(^\circ)</td>
<td>0.302</td>
<td>-1</td>
<td>-17.6(^\circ)</td>
<td>-0.302</td>
</tr>
<tr>
<td>2</td>
<td>37.3(^\circ)</td>
<td>0.606</td>
<td>-2</td>
<td>-37.1(^\circ)</td>
<td>-0.603</td>
</tr>
<tr>
<td>3</td>
<td>65.2(^\circ)</td>
<td>0.908</td>
<td>-3</td>
<td>-65.0(^\circ)</td>
<td>-0.906</td>
</tr>
</tbody>
</table>

On my calculator I get the best straight line fit as

\[
0.302m + 8.33 \times 10^{-4} = \sin \theta_m,
\]

which means that

\[
\lambda = (0.302)(1.73 \mu \text{m}) = 522 \text{ nm}.
\]

E43-4  Although an approach like the solution to Exercise 3 should be used, we'll assume that each measurement is perfect and error free. Then randomly choosing the third maximum,

\[
\lambda = \frac{d \sin \theta}{m} = \frac{(5040 \times 10^{-6} \text{m}) \sin(20.38^\circ)}{3} = 586 \times 10^{-9} \text{m}.
\]

E43-5  (a) The principle maxima occur at points given by Eq. 43-1,

\[
\sin \theta_m = m\frac{\lambda}{d}.
\]

The difference of the sine of the angle between any two adjacent orders is

\[
\sin \theta_{m+1} - \sin \theta_m = (m + 1)\frac{\lambda}{d} - m\frac{\lambda}{d} = \frac{\lambda}{d}.
\]

Using the information provided we can find \( d \) from

\[
d = \frac{\lambda}{\sin \theta_{m+1} - \sin \theta_m} = \frac{(600 \times 10^{-9})}{(0.30) - (0.20)} = 6 \mu \text{m}.
\]

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It doesn't take much imagination to recognize that the second and third order maxima were given.

(b) If the fourth order maxima is missing it must be because the diffraction pattern envelope has a minimum at that point. Any fourth order maxima should have occurred at \( \sin \theta_4 = 0.4 \). If it is a diffraction minima then

\[
a \sin \theta_m = m \lambda \text{ where } \sin \theta_m = 0.4
\]

We can solve this expression and find

\[
a = m \frac{\lambda}{\sin \theta_m} = m \frac{(600 \times 10^{-9} \text{m})}{(0.4)} = m1.5 \mu \text{m}.
\]

The minimum width is when \( m = 1 \), or \( \alpha = 1.5 \mu \text{m} \).

(c) The visible orders would be integer values of \( m \) except for when \( m \) is a multiple of four.

**E43-6**

(a) Find the maximum integer value of \( m = d/\lambda = (930 \text{ nm})/(615 \text{ nm}) = 1.5 \), hence \( m = -1, 0, +1 \); there are three diffraction maxima.

(b) The first order maximum occurs at

\[
\theta = \arcsin(615 \text{ nm})/(930 \text{ nm}) = 41.4^\circ.
\]

The width of the maximum is

\[
\delta \theta = \frac{(615 \text{ nm})}{(1120)(930 \text{ nm}) \cos(41.4^\circ)} = 7.87 \times 10^{-4} \text{ rad},
\]

or \( 0.0451^\circ \).

**E43-7**

The fifth order maxima will be visible if \( d/\lambda \geq 5 \); this means

\[
\lambda \leq \frac{d}{5} = \frac{(1 \times 10^{-9} \text{m})}{(315 \text{ rulings})(5)} = 635 \times 10^{-9} \text{m}.
\]

**E43-8**

(a) The maximum could be the first, and then

\[
\lambda = \frac{d \sin \theta}{m} = \frac{(1 \times 10^{-3} \text{m}) \sin(28^\circ)}{(200)(1)} = 2367 \times 10^{-9} \text{m}.
\]

That's not visible. The first visible wavelength is at \( m = 4 \), then

\[
\lambda = \frac{d \sin \theta}{m} = \frac{(1 \times 10^{-3} \text{m}) \sin(28^\circ)}{(200)(4)} = 589 \times 10^{-9} \text{m}.
\]

The next is at \( m = 5 \), then

\[
\lambda = \frac{d \sin \theta}{m} = \frac{(1 \times 10^{-3} \text{m}) \sin(28^\circ)}{(200)(5)} = 469 \times 10^{-9} \text{m}.
\]

Trying \( m = 6 \) results in an ultraviolet wavelength.

(b) Yellow-orange and blue.
A grating with 400 rulings/mm has a slit separation of
\[ d = \frac{1}{400 \text{ mm}^{-1}} = 2.5 \times 10^{-3} \text{ mm}. \]

To find the number of orders of the entire visible spectrum that will be present we need only consider the wavelength which will be on the outside of the maxima. That will be the longer wavelengths, so we only need to look at the 700 nm behavior. Using Eq. 43-1,
\[ d \sin \theta = m\lambda, \]
and using the maximum angle 90°, we find
\[ m < \frac{d}{\lambda} = \frac{(2.5 \times 10^{-6} \text{ m})}{(700 \times 10^{-9} \text{ m})} = 3.57, \]
so there can be at most three orders of the entire spectrum.

In this case \( d = 2a \). Since interference maxima are given by \( \sin \theta = m\lambda/d \) while diffraction minima are given at \( \sin \theta = m'\lambda/a = 2m'\lambda/d \) then diffraction minima overlap with interference maxima whenever \( m = 2m' \). Consequently, all even \( m \) are at diffraction minima and therefore vanish.

If the second-order spectra overlaps the third-order, it is because the 700 nm second-order line is at a larger angle than the 400 nm third-order line.

Start with the wavelengths multiplied by the appropriate order parameter, then divide both side by \( d \), and finally apply Eq. 43-1.
\[
\begin{align*}
2(700 \text{ nm}) & > 3(400 \text{ nm}), \\
\frac{2}{d} & > \frac{3}{d}, \\
\sin \theta_{2,\lambda=700} & > \sin \theta_{3,\lambda=400},
\end{align*}
\]
regardless of the value of \( d \).

Fig. 32-2 shows the path length difference for the right hand side of the grating as \( d\sin \theta \). If the beam strikes the grating at any angle \( \psi \) then there will be an additional path length difference of \( d\sin \psi \) on the right hand side of the figure. The diffraction pattern then has two contributions to the path length difference, these add to give
\[ d(\sin \theta + \sin \psi) = m\lambda. \]

Let \( d\sin \theta_i = \lambda_i \) and \( \theta_1 + 20^\circ = \theta_2 \). Then
\[ \sin \theta_2 = \sin \theta_1 \cos(20^\circ) + \cos \theta_1 \sin(20^\circ). \]
Rearranging,
\[ \sin \theta_2 = \sin \theta_1 \cos(20^\circ) + \sqrt{1 - \sin^2 \theta_1 \sin(20^\circ)}. \]
Substituting the equations together yields a rather nasty expression,
\[ \frac{\lambda_2}{d} = \frac{\lambda_1}{d} \cos(20^\circ) + \sqrt{1 - (\lambda_1/d)^2} \sin(20^\circ). \]

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Rearranging, 

\[(\lambda_2 - \lambda_1 \cos(20^\circ))^2 = (d^2 - \lambda_1^2) \sin^2(20^\circ)).\]

Use \(\lambda_1 = 430\) nm and \(\lambda_2 = 680\) nm, then solve for \(d\) to find \(d = 914\) nm. This corresponds to 1090 rulings/mm.

**E43-15** The shortest wavelength passes through at an angle of 

\[\theta_1 = \arctan(50\,\text{mm})/(300\,\text{mm}) = 9.46^\circ.\]

This corresponds to a wavelength of

\[\lambda_1 = \frac{(1 \times 10^{-8}\,\text{m}) \sin(9.46^\circ)}{(350)} = 470 \times 10^{-9}\,\text{m}.\]

The longest wavelength passes through at an angle of 

\[\theta_2 = \arctan(60\,\text{mm})/(300\,\text{mm}) = 11.3^\circ.\]

This corresponds to a wavelength of

\[\lambda_2 = \frac{(1 \times 10^{-8}\,\text{m}) \sin(11.3^\circ)}{(350)} = 560 \times 10^{-9}\,\text{m}.\]

**E43-16** (a) \(\Delta \lambda = \lambda/R = \lambda/Nm,\) so

\[\Delta \lambda = (481\,\text{nm})/(620\,\text{rulings/mm})(5.05\,\text{mm})(3) = 0.0512\,\text{nm}.\]

(b) \(m_m\) is the largest integer smaller than \(d/\lambda,\) or

\[m_m \leq 1/(481 \times 10^{-9}\,\text{m})(620\,\text{rulings/mm}) = 3.35,\]

so \(m = 3\) is highest order seen.

**E43-17** The required resolving power of the grating is given by Eq. 43-10

\[R = \frac{\lambda}{\Delta \lambda} = \frac{(589.0\,\text{nm})}{(589.6\,\text{nm}) - (589.0\,\text{nm})} = 982.\]

Our resolving power is then \(R = 1000.\)

Using Eq. 43-11 we can find the number of grating lines required. We are looking at the second-order maxima, so

\[N = \frac{R}{m} = \frac{(1000)}{(2)} = 500.\]

**E43-18** (a) \(N = R/m = \lambda/m\Delta \lambda,\) so

\[N = \frac{(415.5\,\text{nm})}{(2)(415.496\,\text{nm}) - (415.487\,\text{nm})} = 23100.\]

(b) \(d = w/N,\) where \(w\) is the width of the grating. Then

\[\theta = \arcsin \frac{m \lambda}{d} = \arcsin \frac{23100)(2)(415.5 \times 10^{-9}\,\text{m})}{(4.15 \times 10^{-2}\,\text{m})} = 27.6^\circ.\]
We use Eq. 43-12 to first find $d$:

$$d = \frac{m\lambda}{2 \sin \theta} = \frac{(1)(0.261 \times 10^{-9} \text{ m})}{2 \sin(63.8^\circ)} = 1.45 \times 10^{-10} \text{ m}.$$ 

$d$ is the spacing between the planes in Fig. 43-28; it correspond to half of the diagonal distance between two cell centers. Then

$$(2d)^2 = a_0^2 + a_0^2,$$

or

$$a_0 = \sqrt{2d} = \sqrt{2}(1.45 \times 10^{-10} \text{ m}) = 0.205 \text{ nm}.$$ 

Diffraction occurs when $2d \sin \theta = m\lambda$. The angles in this case are then given by

$$\sin \theta = m \frac{(0.125 \times 10^{-9} \text{ m})}{2(0.252 \times 10^{-9} \text{ m})} = (0.248)m.$$ 

There are four solutions to this equation. They are $14.4^\circ$, $29.7^\circ$, $48.1^\circ$, and $82.7^\circ$. They involve rotating the crystal from the original orientation $(90^\circ - 42.4^\circ = 47.6^\circ)$ by amounts

$$47.6^\circ - 14.4^\circ = 33.2^\circ,$$

$$47.6^\circ - 29.7^\circ = 17.9^\circ,$$

$$47.6^\circ - 48.1^\circ = -0.5^\circ,$$

$$47.6^\circ - 82.7^\circ = -35.1^\circ.$$ 

Since the slits are so narrow we only need to consider interference effects, not diffraction effects. There are three waves which contribute at any point. The phase angle between adjacent waves is

$$\phi = 2\pi d \sin \theta / \lambda.$$ 

We can add the electric field vectors as was done in the previous chapters, or we can do it in a different order as is shown in the figure below.

![Diagram](image-url)

Then the vectors sum to

$$E(1 + 2 \cos \phi).$$

We need to square this quantity, and then normalize it so that the central maximum is the maximum. Then

$$I = I_m \frac{(1 + 4 \cos \phi + 4 \cos^2 \phi)}{9}.$$
P43-2 (a) Solve $\phi$ for $I = I_m/2$, this occurs when
\[
\frac{3}{\sqrt{2}} = 1 + 2\cos\phi,
\]
or $\phi = 0.976 \text{ rad}$. The corresponding angle $\theta_x$ is
\[
\theta_x \approx \frac{\lambda\phi}{2\pi d} = \frac{\lambda(0.976)}{2\pi d} = \frac{\lambda}{6.44d}.
\]
But $\Delta\theta = 2\theta_x$, so
\[
\Delta\theta \approx \frac{\lambda}{3.2d}.
\]
(b) For the two slit pattern the half width was found to be $\Delta\theta = \lambda/2d$. The half width in the three slit case is smaller.

P43-3 (a) and (b) A plot of the intensity quickly reveals that there is an alternation of large maximum, then a smaller maximum, etc. The large maxima are at $\phi = 2n\pi$, the smaller maxima are half way between those values.

(c) The intensity at these secondary maxima is then
\[
I = I_m \frac{(1 + 4\cos\pi + 4\cos^2\pi)}{9} = \frac{I_m}{9}.
\]
Note that the minima are not located half-way between the maximal.

P43-4 Covering up the middle slit will result in a two slit apparatus with a slit separation of $2d$. The half width, as found in Problem 41-5, is then
\[
\Delta\theta = \lambda/2(2d) = \lambda/4d,
\]
which is narrower than before covering up the middle slit by a factor of $3.2/4 = 0.8$.

P43-5 (a) If $N$ is large we can treat the phasors as summing to form a flexible “line” of length $N\delta E$. We then assume (incorrectly) that the secondary maxima occur when the loop wraps around on itself as shown in the figures below. Note that the resultant phasor always points straight up. This isn’t right, but it is close to reality.
(a) The direction of propagation is determined by considering the argument of the sine function. As $t$ increases $y$ must decrease to keep the sine function "looking" the same, so the wave is propagating in the negative $y$ direction.

(b) The electric field is orthogonal (perpendicular) to the magnetic field (so $E_x = 0$) and the direction of motion (so $E_y = 0$); Consequently, the only non-zero term is $E_z$. The magnitude of $E$ will be equal to the magnitude of $B$ times $c$. Since $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_0$, when $\mathbf{B}$ points in the positive $x$ direction then $\mathbf{E}$ must point in the negative $z$ direction in order that $\mathbf{S}$ point in the negative $y$ direction. Then

$$E_z = -cB \sin(ky + \omega t).$$

(c) The polarization is given by the direction of the electric field, so the wave is linearly polarized in the $z$ direction.

E44-2 Let one wave be polarized in the $x$ direction and the other in the $y$ direction. Then the net electric field is given by $E^2 = E_x^2 + E_y^2$, or

$$E^2 = E_x^2 \left( \sin^2(kx - \omega t) + \sin^2(kz - \omega t + \beta) \right),$$

where $\beta$ is the phase difference. We can consider any point in space, including $z = 0$, and then average the result over a full cycle. Since $\beta$ merely shifts the integration limits, then the result is independent of $\beta$. Consequently, there are no interference effects.

E44-3 (a) The transmitted intensity is $I_0/2 = 6.1 \times 10^{-3} \text{W/m}^2$: The maximum value of the electric field is

$$E_m = \sqrt{2\mu_0 c I} = \sqrt{2(1.26 \times 10^{-6} \text{H/m})(3.00 \times 10^8 \text{m/s})(6.1 \times 10^{-3} \text{W/m}^2)} = 2.15 \text{ V/m}.$$

(b) The radiation pressure is caused by the absorbed half of the incident light, so

$$p = I/c = (6.1 \times 10^{-3} \text{W/m}^2)/(3.00 \times 10^8 \text{m/s}) = 2.03 \times 10^{-11} \text{Pa}.$$ 

E44-4 The first sheet transmits half the original intensity, the second transmits an amount proportional to $\cos^2 \theta$. Then $I = (I_0/2) \cos^2 \theta$, or

$$\theta = \arccos \sqrt{2I/I_0} = \arccos \sqrt{2(I_0/3)/I_0} = 35.3^\circ.$$

E44-5 The first sheet polarizes the un-polarized light, half of the intensity is transmitted, so $I_1 = \frac{1}{2}I_0$.

The second sheet transmits according to Eq. 44-1,

$$I_2 = I_1 \cos^2 \theta = \frac{1}{2}I_0 \cos^2(45^\circ) = \frac{1}{4}I_0,$$

and the transmitted light is polarized in the direction of the second sheet.

The third sheet is $45^\circ$ to the second sheet, so the intensity of the light which is transmitted through the third sheet is

$$I_3 = I_2 \cos^2 \theta = \frac{1}{4}I_0 \cos^2(45^\circ) = \frac{1}{8}I_0.$$
E44-6 The transmitted intensity through the first sheet is proportional to $\cos^2 \theta$, the transmitted intensity through the second sheet is proportional to $\cos^2(90^\circ - \theta) = \sin^2 \theta$. Then

$$I = I_0 \cos^2 \theta \sin^2 \theta = (I_0/4) \sin^2 2\theta,$$

or

$$\theta = \frac{1}{2} \arcsin \sqrt{4I/I_0} = \frac{1}{2} \arcsin \sqrt{4(0.100I_0)/I_0} = 19.6^\circ.$$

Note that 70.4° is also a valid solution!

E44-7 The first sheet transmits half of the original intensity; each of the remaining sheets transmits an amount proportional to $\cos^2 \theta$, where $\theta = 30^\circ$. Then

$$\frac{I}{I_0} = \frac{1}{2} (\cos^2 \theta)^3 = \frac{1}{2} (\cos(30^\circ))^6 = 0.211.$$

E44-8 The first sheet transmits an amount proportional to $\cos^2 \theta$, where $\theta = 58.8^\circ$. The second sheet transmits an amount proportional to $\cos^2(90^\circ - \theta) = \sin^2 \theta$. Then

$$I = I_0 \cos^2 \theta \sin^2 \theta = (43.3 \text{ W/m}^2) \cos^2(58.8^\circ) \sin^2(58.8^\circ) = 8.50 \text{ W/m}^2.$$

E44-9 Since the incident beam is unpolarized the first sheet transmits 1/2 of the original intensity. The transmitted beam then has a polarization set by the first sheet: 58.8° to the vertical. The second sheet is horizontal, which puts it 31.2° to the first sheet. Then the second sheet transmits $\cos^2(31.2^\circ)$ of the intensity incident on the second sheet. The final intensity transmitted by the second sheet can be found from the product of these terms,

$$I = (43.3 \text{ W/m}^2) \left(\frac{1}{2}\right) (\cos^2(31.2^\circ)) = 15.8 \text{ W/m}^2.$$

E44-10 $\theta_p = \arctan(1.53/1.33) = 49.0^\circ$.

E44-11 (a) The angle for complete polarization of the reflected ray is Brewster’s angle, and is given by Eq. 44-3 (since the first medium is air)

$$\phi_p = \tan^{-1} n = \tan^{-1}(1.33) = 53.1^\circ.$$

(b) Since the index of refraction depends (slightly) on frequency, then so does Brewster’s angle.

E44-12 (b) Since $\theta_r + \theta_p = 90^\circ$, $\theta_p = 90^\circ - (31.8^\circ) = 58.2^\circ$.

(a) $n = \tan \theta_p = \tan(58.2^\circ) = 1.61$.

E44-13 The angles are between

$$\theta_p = \tan^{-1} n = \tan^{-1}(1.472) = 55.81^\circ.$$

and

$$\theta_p = \tan^{-1} n = \tan^{-1}(1.456) = 55.52^\circ.$$

E44-14 The smallest possible thickness $t$ will allow for one half a wavelength phase difference for the o and e waves. Then $\Delta n t = \lambda/2$, or

$$t = (525 \times 10^{-9} \text{ m})/2(0.022) = 1.2 \times 10^{-5} \text{ m}.$$
E44-15 (a) The incident wave is at 45° to the optical axis. This means that there are two components; assume they originally point in the +y and +z direction. When they travel through the half wave plate they are now out of phase by 180°; this means that when one component is in the +y direction the other is in the −z direction. In effect the polarization has been rotated by 90°.

(b) Since the half wave plate will delay one component so that it emerges 180° "later" than it should, it will in effect reverse the handedness of the circular polarization.

(c) Pretend that an unpolarized beam can be broken into two orthogonal linearly polarized components. Both are then rotated through 90°; but when recombed it looks like the original beam. As such, there is no apparent change.

E44-16 The quarter wave plate has a thickness of \( z = \lambda/4\Delta n \), so the number of plates that can be cut is given by
\[
N = (0.250 \times 10^{-3} \text{m})4(0.181)/(488 \times 10^{-9} \text{m}) = 371.
\]

P44-1 Intensity is proportional to the electric field squared, so the original intensity reaching the eye is \( I_0 \), with components \( I_h = (2.3)^2 I_v \), and then
\[
I_0 = I_h + I_v = 6.3I_v, \quad \text{or} \quad I_v = 0.16I_0.
\]

Similarly, \( I_h = (2.3)^2 I_v = 0.84I_0 \).

(a) When the sun-bather is standing only the vertical component passes, while

(b) when the sun-bather is lying down only the horizontal component passes.

P44-2 The intensity of the transmitted light which was originally unpolarized is reduced to \( I_u/2 \), regardless of the orientation of the polarizing sheet. The intensity of the transmitted light which was originally polarized is between 0 and \( I_p \), depending on the orientation of the polarizing sheet. Then the maximum transmitted intensity is \( I_u/2 + I_p \), while the minimum transmitted intensity is \( I_u/2 \). The ratio is 5, so
\[
5 = \frac{I_u/2 + I_p}{I_u/2} = 1 + \frac{2I_p}{I_u},
\]

or \( I_p/I_u = 2 \). Then the beam is 1/3 unpolarized and 2/3 polarized.

P44-3 Each sheet transmits a fraction
\[
\cos^2 \alpha = \cos^2 \left( \frac{\theta}{N} \right).
\]

There are \( N \) sheets, so the fraction transmitted through the stack is
\[
\left( \cos^2 \left( \frac{\theta}{N} \right) \right)^N.
\]

We want to evaluate this in the limit as \( N \to \infty \).

As \( N \) gets larger we can use a small angle approximation to the cosine function,
\[
\cos x \approx 1 - \frac{1}{2}x^2 \quad \text{for} \quad x \ll 1
\]

The the transmitted intensity is
\[
\left( 1 - \frac{1}{2} \frac{\theta^2}{N^2} \right)^{2N}.
\]
This expression can also be expanded in a binomial expansion to get
\[ 1 - 2N \frac{1}{2} \frac{\theta^2}{N^2}, \]
which in the limit as \( N \to \infty \) approaches 1.

The stack then transmits all of the light which makes it past the first filter. Assuming the light is originally unpolarized, then the stack transmits half the original intensity.

**P44-4**  
(a) Stack several polarizing sheets so that the angle between any two sheets is sufficiently small, but the total angle is 90°.

(b) The transmitted intensity fraction needs to be 0.95. Each sheet will transmit a fraction \( \cos^2 \theta \), where \( \theta = 90°/N \), with \( N \) the number of sheets. Then we want to solve
\[ 0.95 = \left( \cos^2(90°/N) \right)^N \]
for \( N \). For large enough \( N \), \( \theta \) will be small, so we can expand the cosine function as
\[ \cos^2 \theta = 1 - \sin^2 \theta \approx 1 - \theta^2, \]
so
\[ 0.95 \approx \left( 1 - \left( \pi/2N \right)^2 \right)^N \approx 1 - N\left(\pi/2N\right)^2, \]
which has solution \( N = \pi^2/4(0.05) = 49 \).

**P44-5**  
Since passing through a quarter wave plate twice can rotate the polarization of a linearly polarized wave by 90°, then if the light passes through a polarizer, through the plate, reflects off the coin, then through the plate, and through the polarizer, it would be possible that when it passes through the polarizer the second time it is 90° to the polarizer and no light will pass. You won’t see the coin.

On the other hand if the light passes first through the plate, then through the polarizer, then is reflected, the passes again through the polarizer, all the reflected light will pass through he polarizer and eventually work its way out through the plate. So the coin will be visible.

Hence, side \( A \) must be the polarizing sheet, and that sheet must be at 45° to the optical axis.

**P44-6**  
(a) The displacement of a ray is given by
\[ \tan \theta_k = y_k/t, \]
so the shift is
\[ \Delta y = t(\tan \theta_e - \tan \theta_o). \]

Solving for each angle,
\[ \theta_e = \arcsin \left( \frac{1}{1.486} \sin(38.8°) \right) = 24.94°, \]
\[ \theta_o = \arcsin \left( \frac{1}{1.658} \sin(38.8°) \right) = 22.21°. \]

The shift is then
\[ \Delta y = (1.12 \times 10^{-2} \text{m}) (\tan(24.94°) - \tan(22.21°)) = 6.35 \times 10^{-4} \text{m}. \]

(b) The \( e \)-ray bends less than the \( o \)-ray.
(c) The rays have polarizations which are perpendicular to each other; the \( o \)-wave being polarized along the direction of the optic axis.
(d) One ray, then the other, would disappear.
P44-7  The method is outline in Sample Problem 44-24; use a polarizing sheet to pick out the o-ray or the e-ray.