

UNIT EQUATION APPROACH TO PROBLEM SOLVING

Conversion Factors

Conversion factors are ratios with units. When we saw that 2.54 cm is equal to 1 inch (by definition) then we can express that as either of these conversion factors:

$$\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right) = \left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right)$$

Because we know that algebraically, any value over itself is equal to one, these two conversions are equal to each other. Furthermore, a term or equation can be multiplied by one and still be true, thus a term or equation can be multiplied by a conversion factor, and still be true (because a conversion factor is equal to one). Therefore we can convert units by applying conversion factors. For example we can convert 543.1 cm to inches by:

$$(543.1 \text{ cm})\left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right) = 213.82 \text{ in} = 213.8 \text{ in}$$

How did we know which of the two conversion factors to apply? If we look at the units, we started with cm and want to cancel them. Because anything over itself, including units, is equal to one and would cancel out of a factor, by putting cm in the denominator, cm would cancel out of the expression. Furthermore, we want our answer to have the units of inches, and thus must put those units in the numerator.

Notice the significant figures in this example. Why are there 4 SF's? In this example, the conversion factor does not limit the number of SF's because it is a definition. An inch is defined as exactly 2.54 cm, and thus has an infinite number of SF's (even though we are too lazy to show that many in our conversion factor.) In this example the SF's are limited by the value we are converting. It should be noted that this is not always the case as not all conversion factors are exact numbers.

Exact numbers should be used in conversion factors, whenever possible. The numbers in the following definitions are exact, and thus have infinite SF's.

1 in = 2.54 cm	1 mL = 1 cm ³	1 gal = 231 in ³	1 gal = 4 qt
1 qt = 2 pt	1 pt = 16 fluid oz	1 metric ton = 10 ³ kg	1 lb = 453.59237 g
1 lb = 16 oz	9 °F = 5 °C	1 N (Newton) = 10 ⁵ dyne	
1 J = 10 ⁷ erg	1 cal = 4.184 J		

Unit Equation Approach

Problem solving may require applying a series of conversion factors to achieve the desired result. When this is necessary, set the conversion factors in a logical sequence, one after the other. The result should be a single-lined unit equation that is easy to read and understand. For example, to convert 0.06250 km to feet:

$$(0.06250 \text{ km}) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right) \left(\frac{10^2 \text{ cm}}{1 \text{ m}} \right) \left(\frac{1 \text{ in}}{2.54 \text{ cm}} \right) \left(\frac{1 \text{ ft}}{12 \text{ in}} \right) = 205.1052 \text{ ft} = 205.1 \text{ ft}$$

Notice the logical sequence of conversion factors: km are converted to m, m to cm, cm to in and in to ft. Because all of these conversion factors are definitions, and therefore exact numbers with infinite SF's, the SF's for the answer is determined by the original value we are converting.

Practice doing unit conversion by the unit equation approach by doing the following. You'll find the answers below.

1. Convert 0.8259 in^2 to square millimeters.
2. How many gallons are in 0.4308 ft^3 ?
3. Change 8.55 g/mL to kilogram per cubic meter.

Density can be thought of as a conversion factor between the mass of a sample and its volume. Thus density, and other physical constants, can be applied to values in the same way as we have been doing here. For example, the volume (ft^3) of $3.09 \times 10^3 \text{ g}$ of aluminum, which has a density of 2.70 g/mL , can be calculated by:

$$(3.09 \times 10^3 \text{ g}) \left(\frac{1 \text{ mL}}{2.70 \text{ g}} \right) \left(\frac{1 \text{ cm}^3}{1 \text{ mL}} \right) \left(\frac{1 \text{ in}}{2.54 \text{ cm}} \right)^3 \left(\frac{1 \text{ ft}}{12 \text{ in}} \right)^3 = 0.04041 \text{ ft}^3 = \boxed{0.0404 \text{ ft}^3}$$

Answers:

$$1. (0.8259 \text{ in}^2) \left(\frac{2.54 \text{ cm}}{1 \text{ in}} \right)^2 \left(\frac{10 \text{ mm}}{1 \text{ cm}} \right)^2 = 532.83 \text{ mm}^2 = 532.8 \text{ mm}^2$$

Pay attention to the fact that in this problem, the conversion factors have to be squared. Because the original value being converted is in in^2 , meaning in x in, we have to convert both in to cm and then to mm.

$$2. (0.4308 \text{ ft}^3) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)^3 \left(\frac{1 \text{ gal}}{231 \text{ in}^3} \right) = 3.2226 \text{ gal} = 3.223 \text{ gal}$$

Notice that in this problem, the first conversion factor has to be cubed because we start with ft^3 , but the second does not because gallons are related to cubic inches in that conversion factor.

$$3. \left(\frac{8.55 \text{ g}}{\text{mL}} \right) \left(\frac{1 \text{ kg}}{10^3 \text{ g}} \right) \left(\frac{1 \text{ mL}}{1 \text{ cm}^3} \right) \left(\frac{10^2 \text{ cm}}{1 \text{ m}} \right)^3 = 855.0 \text{ kg/m}^3$$

This problem requires the conversion of units for the numerator and the denominator, but this is handled in exactly the same way. Simply pay attention to what units you are trying to cancel, and set the conversion factor so that it puts those units in the opposite position.

Unit Conversion and Density Worksheet

A) Complete the following conversions (remember significant figures!)

- 1) 2.0 m to cm
- 2) 0.250 L to mL
- 3) 5.00 cm to km
- 4) 3.00 kg to mg
- 5) 6.00 qt to L
- 6) 3.00 g to lb
- 7) 5.00 cm to in
- 8) 325 mL to qt
- 9) 5.280×10^3 ft to m
- 10) 2.0 yr to sec
- 11) \$0.90/gal to \$/L
- 12) 196 K to °C
- 13) 98.6 °F to °C
- 14) 0.00 K to °F

B) Complete the following problems:

- 15) 23.9 g of water at 100 °C occupy a volume of 25.0 mL. What is the density of water?
- 16) If the density of gasoline is 0.70 g/mL, what is the mass in g of 50. L of gasoline?
- 17) The density of acetic acid is 1.05 g/mL. How many mL do you need to provide 125 g of acetic acid?
- 18) A cylinder of titanium (length = 5.00 cm, diameter = 1.20 cm) has a mass of 25.5 g. What is the density of titanium?
- 19) The density of iron is 7.86 g/cm³. What is the mass of a piece of iron that raises the level of water from the 18.3 mL mark to the 24.8 mL mark, when it is placed in a graduated cylinder?
- 30) The density of lead is 11.3 g/cm³. What is the volume of 25 g of lead?

Significant Figures
all of the numbers in a measurement
(the ones we are certain about, plus the first uncertain number)

Rules for counting SF:

- 1) Some numbers have infinite number of SF
 - counted numbers 89 students
 - definitions 1000 m = 1 km
 - "exact numbers" 2.54 cm = 1 in
- 2) Nonzero integers are always significant: 1,256 = 4 SF's
- 3) Zeros:
 - a) Captured zeros (between other digits) are always significant: 1006 = 4 SF's
 - b) Leading zeros (preceding other digits) are never significant: 0.039 = 2 SF's
 - c) Trailing zeros (to the right of other digits) can be ambiguous. Trailing zeroes are significant only if the value includes a decimal point, or a dotted line separating significant from nonsignificant digits:

1,420 = 3 SF's	1,420. = 4 SF's	1,410.0 = 5 SF's
10,000 = 1 SF's	10:000 = 2 SF's	10,0:00 = 3 SF's
10,00:0 = 4 SF's	10,000: = 5 SF's	10,000.0 = 6 SF's

Rules for calculating with SF:

Rule 1) Addition, subtraction, averages: Look at the decimal place.

$$\begin{array}{r} 251.322 \\ + 3.1 \\ \hline 254.4:22 = 254.4 \end{array}$$

Rule 2) Multiplying or dividing: Count the number of SF

$$(2.5631)(1.12) = 2.870672 = 2.87$$

Rules for rounding off numbers:

- 1) If the digit to be dropped is less than 5, simply drop the digit.
8.92:1 = 8.92
- 2) If the digit to be dropped is greater than 5, increase the preceding digit by one.
7.45:62 = 7.46
- 3) If the digit to be dropped is EXACTLY 5, then either drop it or increase the preceding digit by one, to make the last SF an EVEN number.
5.32:5 = 5.32
67.1:500 = 67.2

Rules for setting up calculations:

- 1) Indicate what is being calculated
- 2) Show the equation
- 3) Proceed with the calculation in a logical sequence.
- 4) Express intermediate answers with one non-significant digit (use a dashed line.)
- 5) Box the final answer with the appropriate number of SF's and units.

SIGNIFICANT FIGURE WORKSHEET

1) Indicate the number of significant figures in the following:

- a) 2.508 L _____ d) 12 in/1 ft _____ g) 29 mm _____
 b) 0.00210 km _____ e) 20 students _____ h) 3.50×10^3 nm _____
 c) 3000 g _____ f) 45.00 g _____ I) 8.34:5 g/mL _____

2) Round each of the following numbers to the indicated number of significant figures:

- a) 96.528 cm² to 3 SF _____ d) 0.036559 m to 4 SF _____
 b) 1275 mL to 2 SF _____ e) 0.14500001 lb to 2 SF _____
 c) 26.485 g to 4 SF _____ f) 6.0221367×10^{23} to 4 SF _____

3) What is the assumed absolute uncertainty (au) of the following?

- a) 0.0770 g _____ d) 1.56×10^3 m _____
 b) 450 mL _____ e) 4500 km _____
 c) 35:14 cm² _____ f) 25.0 lb _____

4) Perform the following calculations, giving the answer properly rounded to the correct number of significant figures.

- a) $65\text{g} + 25.86\text{g} + 145.1\text{g} =$ _____ c) $2.5 \times 10^2 + 2.6 \times 10^3 + 3.4 \times 10^{-1} =$ _____
 b) $415.1\text{g} - 25.56\text{g} =$ _____ d) $\frac{4.582\text{g} + 4.54\text{g} + 4.60\text{g}}{3} =$ _____

5) What is the assumed relative uncertainty (ru) of the following?

- a) 0.0770 g _____ d) 1.56×10^3 m _____
 b) 450 mL _____ e) 4500 km _____
 c) 35:14 cm² _____ f) 25.0 lb _____

6) Perform the following calculations, giving the answer properly rounded to the correct number of significant figures.

- a) $6.72\text{m} \left(\frac{1\text{mm}}{10^{-3}\text{m}} \right) =$ _____ c) $\frac{(8.015 \times 10^{-3})(0.550)}{1.9 \times 10^3} =$ _____
 b) $\frac{60.00 \times 35}{0.0050} =$ _____ d) $(22.5\text{cm})(125.6\text{cm})(1.4\text{cm}) =$ _____

7) Perform the following calculations, giving the answer properly rounded to the correct number of significant figures. Hint: You might want to show an intermediate answer.

- a) $\frac{35.46\text{g/mol} - 35.4527\text{g/mol}}{35.4527\text{g/mol}} \times 100\% =$ _____
 b) $Q = 50.0\text{g} \left(\frac{2.09\text{J}}{\text{g}^\circ\text{C}} \right) (28.5^\circ\text{C} - 26.2^\circ\text{C}) =$ _____

Scientific Notation, Significant Figures & Precision and Accuracy**A) Use scientific notation to express each of the following:**

- | | | |
|------------|----------|-----------|
| 1) 3.14 | 4) 150 | 7) 22.4 |
| 2) 0.08206 | 5) 0.150 | 8) 0.003 |
| 3) 150.0 | 6) 8314 | 9) 0.0030 |

B) How many significant figures are in each of the following?

- | | | |
|------------|------------|----------------------------|
| 10) 24.305 | 13) 0.726 | 16) 0.81 |
| 11) 260 | 14) 800 | 17) 0.082060 |
| 12) 47.90 | 15) 14.007 | 18) 1.620×10^{-2} |

C) Express each of the following with 3 significant figures:

- | | | |
|----------------------------|------------|----------------------------|
| 19) 3.14159 | 22) 1.0080 | 25) 35.453 |
| 20) 0.08206 | 23) 15.994 | 26) 1.625×10^{-2} |
| 21) 6.022×10^{23} | 24) 0.9983 | 27) 150.4 |

D) Complete the following calculations. Be sure to express your answer with the correct number of significant figures.

- | | |
|--|--|
| 28) $(12.0)(2.54) =$ | 34) $3.2 + 14.19 =$ |
| 29) $0.250/1.057 =$ | 35) $3.41 \times 10^4 + 6.3 \times 10^3 =$ |
| 30) $\frac{453.1 + 2.102}{16.0} =$ | 36) $\frac{7.2 \times 10^{-4}}{8.0 \times 10^2} =$ |
| 31) $\frac{3.24 \times 10^{22}}{6.023 \times 10^{23}} =$ | 37) $4.8 \times 10^3 / 7.99 \times 10^{-2} =$ |
| 32) $1.00 / 6.023 \times 10^{23} =$ | 38) $7.70 \times 10^{-1} / 1.1 \times 10^3 =$ |
| 33) $(6.023 \times 10^{23})(2.5 \times 10^{-4}) =$ | 39) $\frac{(5.4 \times 10^2)(7.99 \times 10^{-3})}{(4.0 \times 10^1)(9 \times 10^{-5})} =$ |

E) The mass of a 5.000 gram piece of iron is measured 3 times on 2 different balances with the following results:

Trial	Balance 1 (g)	Balance 2 (g)
1	5.01	4.97
2	4.99	4.88
3	5.02	4.91

40) What is the average for balance 1? For balance 2?

41) What is the average deviation for balance 1? For balance 2?

42) Based on these measurements, which balance is more accurate? Which is more precise?