EXPERIMENTAL ERRORS OR UNCERTAINTIES

The question of what constitutes the most reliable value to be assigned as the uncertainty of any given measured quantity is one that has been discussed for many decades and, presumably, will continue to be discussed. It is a question that involves many considerations and by its very nature has no unique answer. The subject of the propagation of error, on the contrary, is a purely mathematical matter, with very definite and easily determined conclusions.

When computing a result by combining several measurements, the errors of the measurements will obviously lead to the result having a certain range of error about the computer value of the result. Propagation of error by means of error equations is a mathematical method of determining how the errors in the terms carry through and affect the result.

The theory of the propagation of errors shows also what the important errors are (in the measured quantities and computed results) and what must be done to perform a better experiment (purchase better equipment or be more careful). There is no advantage of expending time and resources on improving the accuracy of some value if the if equations for the propagation of errors show that there are other factors influencing the result.

I Classification of Errors

A Systematic Errors: Errors in the accuracy of results (errors in instruments calibration.)

1 Calibration of instruments has certain limits of accuracy and experimental conditions (i.e. temperature) may differ from the calibration conditions.
2 Personal errors—such as systematic parallax introduced by personal habits when taking readings on instruments.
3 Poor technique in operation of equipment.

B Random Errors: Errors in the precision of results (errors in the measured values.)

1 Errors in estimating fraction of smallest division on the scale being used. (reading error)
2 Small disturbances or fluctuating experimental conditions.
3 Actual randomness of measured quantity, i.e. "diameter" of a wire depends where along the wire it is measured.

C Errors that should not be present and cause for repeating the experiment.

1 Blunders—can sometimes be avoided by trial computations and use of logic.
2 Errors of computation.
3 Large disturbances.

II Propagation of Errors:

Let the computed quantity \( Q \) be a function of the measured quantities \( x, y, \ldots \); \( Q = f(x, y, \ldots) \) which may be an addition, multiplication, etc., operation. Given uncertainties \( \Delta x, \Delta y, \ldots \), what is the resulting uncertainty \( \Delta Q \) in \( Q \)? These uncertainties or "errors" are referred to as the absolute errors. The following relationships are determined by applying the Taylor expansion to the computed function.
A **Addition and/or Subtraction:**

The error in a sum or difference is determined from

If \( Q = x + y \) or \( Q = x - y \)

Then the error result is: \( \Delta Q = \Delta x + \Delta y \)

Thus the absolute errors add.

B **Simple product or quotient:** When quantities are multiplied or divided, the relative or percent errors add.

If \( Q = \frac{A}{B} \) or \( Q = \frac{A}{C} \)

Then

\[
\frac{\Delta Q}{Q} = \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}
\]

Note that the terms \( A, B \), could be the result of a sum or difference or measured quantities \( x, y, z \), in which case, the absolute error of those terms would be determined by the first rule, i.e. if \( A = x - y \), then \( \Delta A = \Delta x + \Delta y \) or if

\[
\frac{Q}{Q} = \frac{(x - y)(B)}{C}
\]

Then

\[
\frac{\Delta Q}{Q} = \frac{\Delta x + \Delta y}{x - y} + \frac{\Delta B}{B} + \frac{\Delta C}{C}
\]

The terms \( \Delta A/\text{A etc.} \) are referred to as the relative errors.

C **Product of factors raised to powers:** The relative error of each factor is multiplied by the exponent of that factor.

If

\[
Q = \frac{A^2 B^3}{C^3}
\]

then the error equation is

\[
\frac{\Delta Q}{Q} = \frac{2\Delta A}{A} + \frac{1 \Delta B}{B} + \frac{3\Delta C}{C}
\]

Again in the \( A, B, \) or \( C \) are the result of a sum or difference, then \( \Delta A \) etc., would be handled by the first rule.

III The next step is to find the size of the errors associated with the measurements.

A The random error can be estimated by one of several methods.

1 Reading error can be used as the random error and usual practice is to use one-half of the least count of instrument scale. This is not a strict rule and can be modified by your knowledge of the instrument and measuring.

2 The standard deviation or the probable error of the mean can be used as the random error if the measurements are analyzed statistically. A best fit curve determined by computer analysis is an example.

B The calibration error is usually given with the instrument; but in our case the necessary values are posted in the lab. These are given as a percent error or as an absolute error.
IV Total Error

Now with the total error of each measurement given by the sum of the random error and the calibration error you are ready to calculate the error of the result by use of the error equation. Then the final answer would be the calculated value of the result ± the error as the result.

A When asked to compare your result to an accepted value, the best procedure is to calculate the percent error by:

\[
\%Error = \frac{(Result - Accepted)}{Accepted} \times 100\%
\]

B When comparing two experimental values of the same thing, you should use the % difference method:

\[
\%Difference = \frac{(1^{st} value - 2^{nd} value)}{average} \times 100\%
\]

Where the average is \(\frac{1}{2} (1^{st} value + 2^{nd} value)\)

The above are the general rules for determining the number of digits in an experimental result. However, for most cases there are special short-cut methods that may be used for an approximation. If the above general rules would not be used in your particular case, then the laboratory instructor would explain these short-cut methods in class. However, see the note below about the number of digits to be kept in a computerized result.

**Note:** In order to determine the number of digits in a result obtained by a computerized FIT program, use the standard deviation or root mean square error as computed by computer along with the best-fit equation, unless error analysis is required.
ERROR ANALYSIS WORKSHEET

Fill in the corresponding tables as appropriate for determining the error $\Delta Q$ of the dependent quantity $Q$ due to experimental uncertainties.

## 1

<table>
<thead>
<tr>
<th>Measured Quantity—Name, Symbol, and Value</th>
<th>Reading Error</th>
<th>Calibration Error</th>
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<tbody>
<tr>
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<td>Absolute</td>
<td>Relative %</td>
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## 2

List in this table the FORMULAE and ERROR PROPAGATION EQUATIONS, and also evaluate error, then tabulate the results in Table 3, or Table 4 as appropriate.

Example:

$$S = S_0 + V_0 t$$

$$\Delta S = \Delta S_0 + \Delta V_0 + V_0 \Delta t = 0.02 + 3.01 (0.05) + 7.12 (0.01) = 0.2447 = 0.2 \text{ cm}$$

<table>
<thead>
<tr>
<th>Formula for $Q$</th>
<th>Error Equation and Value for $\Delta Q$</th>
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