Exercises:

1. \( F = 100 \text{N} \)
   \( L = 4.00 \text{ m} \)

   a) \( \overline{F} = FL \) out of page
      \( \overline{T} = 10 \text{ (4)} \) out of page
      \( \overline{T} = 40.0 \text{ N} \cdot \text{m} \) out of page

   b) \( \overline{F} = F \cos(30^\circ) L \) out of page
      \( \overline{T} = 10 \cos(30^\circ) \text{ (4)} \) out of page
      \( \overline{T} = 34.6 \text{ N} \cdot \text{m} \) out of page

   c) \( \overline{F} = F \sin(30^\circ) L \) out of page
      \( \overline{T} = 10 \sin(30^\circ) \text{ (4)} \) out of page
      \( \overline{T} = 20.0 \text{ N} \cdot \text{m} \) out of page

   d) \( \overline{F} = F \sin(60^\circ) \) (2) into page
      \( \overline{T} = 10 \sin(60^\circ) \text{ (2)} \) into page
      \( \overline{T} = 17.3 \text{ N} \cdot \text{m} \) into page

   e) \( \overline{T} = 0 \)

   f) \( \overline{T} = 0 \)
\[ T_1 = m_1 a \]
\[ T_1 = 12a \quad \text{eqn #1} \]

\[ \sum F_y = m a \]
\[ m_2 g - T_2 = m_2 a \]
\[ 5(9.8) - T_2 = 5a \]
\[ 49 - T_2 = 5a \quad \text{eqn #2} \]
\[ T_2 = 49 - 5a \]

\[ (\sum \tau)_{\text{mom}} = I \alpha \]
\[ T_2 R - T_1 R = \frac{1}{2} MR^2 \alpha \]
\[ T_2 - T_1 \]
\[ R = \frac{1}{2} MR^2 \left( \frac{a}{R} \right) \]
\[ T_2 - T_1 = \frac{1}{2} (a) a \]
\[ T_2 - T_1 = a \quad \text{eqn #3} \]
\[ (49 - 5a) - 12a = a \]
\[ 49 = 18a \]
\[ a = 2.72 \frac{m}{s^2} \]

\[ T_1 = 12a \text{ and } T_2 = 49 - 5(2.72) \]
\[ T_1 = 32.7 N \]
\[ T_2 = 35.4 N \]
c) Apply $\Sigma F = m \vec{a}_c$ to the pulley.

\[
\frac{1}{2} \Sigma F_x = m a_x \\
N_x - T_1 = 2 (0) \\
N_x = T_1 = 32.7 \text{ N}
\]

\[
\Sigma F_y = m a_y \\
N_y = M g - T_2 = M (0) \\
N_y = 2 (9.8) - 35.4 = 0 \\
N_y = 55.0 \text{ N}
\]

\[\text{Hoop, } m = 0.180 \text{ kg} \]

\[R = 8 \text{ cm} = 0.08 \text{ m} \]

\[h = 75 \text{ cm} = 0.75 \text{ m} \]

\[v_0 = 0 \text{ and } w_0 = 0.\]

\[E_i + \Sigma W_{nc} = E_f \\
= m g h + \frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 + W_T = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 \\
= m g h = \frac{1}{2} m v_f^2 + \frac{1}{2} (M R^2) (\frac{v}{R})^2 \\
\text{since } w = \frac{v}{R} \text{ and } I = M R^2 \\
g h = v_f^2 \Rightarrow v_f = \sqrt{g h} = \sqrt{9.8 (0.75)}
\]

\[v_f = 2.71 \text{ m/s} \]

Thus

\[W_f = \frac{v_f}{R} = \frac{2.71}{0.08} \]

\[W_f = 33.9 \text{ rad/s} \]

\[3\]
26  

(a) See free body diagram. The angular speed of the ball is decreasing so the angular acceleration is counterclockwise. The net torque must therefore be counterclockwise about the cm. The frictional force must act uphill to provide the counterclockwise torque about the center of mass.

\[ \sum F_x = m a_x \]
\[ mg \sin \beta - f_s = m a_{cm} \]
\[ \sum \tau_{cm} = I_{cm} \alpha \]
\[ f_s R = \frac{2}{5} MR^2 (\theta_{cm}) \]
\[ f_s = \frac{2}{5} m a_{cm} \]

\[ mg \sin \beta - \frac{2}{5} R \theta_{cm} = m a_{cm} \]
\[ g \sin \beta = \frac{2}{5} a_{cm} + a_{cm} \]
\[ g \sin \beta = \frac{7}{5} a_{cm} \]
\[ a_{cm} = \frac{5}{7} g \sin \beta \]

(b) \[ \sum F_y = m g \]
\[ n - mg \cos \beta = 0 \]
\[ n = mg \cos \beta \]

(c) \[ f_s = \frac{2}{5} m a_{cm} \]
\[ f_s = \frac{2}{5} m (\frac{5}{7} g \sin \beta) \]
\[ f_s = \frac{2}{7} mg \sin \beta \]

\[ f_s \leq \mu_s n \]
\[ \frac{3}{7} mg \sin \beta \leq \mu_s mg \cos \beta \]
\[ \mu_s \geq \frac{3}{7} \tan \beta \]
\[ \mu_s \text{ min} = \frac{3}{7} \tan \beta \]
L = 2.08 m
m = 117 kg
\tau = 1950 \text{ N.m}
\nu_0 = \nu_0 = 0

\text{I}_e = \frac{1}{12} m L^2
\text{I}_e = \frac{1}{12} (117)(2.08)^2
\text{I}_e = 42.2 \text{ kg.m}^2

a) \quad \theta = \frac{1}{\text{I}_e} \int \text{I}_e \, \tau \, d\theta
\quad 1950 = 42.2 \alpha
\quad \alpha = 46.2 \text{ rad sec}^{-2}

b) \quad \Delta \theta = 5 \text{ rev} \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 31.416 \text{ rad}
\quad \omega_f^2 = \omega_0^2 + 2 \alpha \Delta \theta
\quad \omega_f^2 = 0 + 2 (46.2)(31.416)
\quad \boxed{\omega_f = 53.9 \text{ rad sec}}

c) \quad \mathbf{W} = \tau \Delta \theta
\quad \mathbf{W} = 1950 (31.416)
\quad \mathbf{W} = 6.13 \times 10^4 \text{ J}

d) \quad \mathbf{P}_{\text{ave}} = \frac{\mathbf{Work}}{\text{time}} = \frac{\tau \Delta \theta}{\Delta t} = \tau \Delta \theta = \frac{1}{2} (\omega_0^2 + \omega_f^2)
\quad \mathbf{P}_{\text{ave}} = \frac{1}{2} \tau \omega_f = \frac{1}{2} (1950)(53.9) \Rightarrow \mathbf{P}_{\text{ave}} = 5.25 \times 10^5 \text{ Watts}

\text{Alternatively,} \quad \mathbf{Work} = \Delta (kE) \quad \text{and} \quad \omega_f = \omega_0 + \alpha t
\quad \Delta \theta = \frac{1}{2} \text{I}_e \omega_f^2
\quad \omega_f = \frac{1}{2} (42.2)(53.9)^2
\quad \mathbf{Work} = 6.13 \times 10^4 \text{ J}
\quad \mathbf{P}_{\text{ave}} = \frac{\mathbf{Work}}{\text{time}} = \frac{6.13 \times 10^4 \text{ J}}{1.167 \text{ sec}} \Rightarrow \mathbf{P}_{\text{ave}} = 5.25 \times 10^4 \text{ Watts}
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37. \( \vec{T}_0 = \vec{r} \times \vec{p} \)

\[ \vec{p} = -8 \sin(36.9) \hat{i} + 8 \cos(36.9) \hat{j} \]
\[ \vec{r} = (-6.397 \hat{i} + 4.803 \hat{j}) \]
\[ \vec{p} = m \vec{v} = 2 (12z) = 24.0 \text{ kg} \cdot \text{m} \cdot \text{sec} \]

\[ \vec{T}_0 = (6.397 \hat{i} + 4.803 \hat{j}) \times (24 \hat{k}) \]

\[ \vec{T}_0 = 115 \text{ kg} \cdot \text{m} \cdot \text{sec} \]

\[ \vec{T}_0 \text{ into the page} \]

\[ b) \quad \vec{\xi} = \frac{d\vec{T}_0}{dt} \quad \text{but} \quad \vec{\xi} = Mg \left[ \cos(36.9) \right] \text{ counterclockwise} \]

\[ 2(9.8)(8) \cos(36.9) \hat{k} = \frac{d\vec{T}_0}{dt} \]

\[ \frac{d\vec{T}_0}{dt} = (125 \text{ N} \cdot \text{m}) \text{ out of the page} \]

40. \text{ Sphere (spherical shell)} \quad \pi \quad \text{a)} \quad I_z = \frac{2MB^3}{3} \quad \text{b)} \quad t = 3 \text{ sec} \quad \text{c)} \quad L = I \omega \]

\[ \omega = \frac{d\theta}{dt} = (3t + 4.4t^3) \]

\[ L = 0.4608 \left[ 3(3) + 4.4(3^3) \right] \]

\[ L = 58.9 \text{ kg} \cdot \text{m}^2 \cdot \text{sec} \]

\[ \tau = I \frac{d\omega}{dt} \quad \text{where} \quad \tau = 3 + 13.2 t^2 \]

\[ \tau = (0.4608)(121.8) \]

\[ \tau = 56.1 \text{ N} \cdot \text{m} \]
\[ m = 0.025 \text{ kg} \]
\[ r_0 = 0.3 \text{ m} \]
\[ \omega_0 = 1.75 \text{ rad/}\text{sec} \]
\[ r_f = 0.15 \text{ rad/}\text{sec} \]

a) Yes, the angular momentum of the block is conserved.

b) \[ \sum \tau = I \Omega \text{ conservation of angular momentum} \]
\[ I_0 \omega_0 = I_f \omega_f \]
\[ m r_0^2 \omega_0 = m r_f^2 \omega_f \]
\[ (0.3)^2 (1.75) = (0.15)^2 \omega_f \]
\[ \omega_f = 7.00 \text{ rad/}\text{sec} \]

\[ \Delta (kE) = kE_f - kE_i \]
\[ = \frac{1}{2} I \omega_f^2 + \frac{1}{2} I \omega_i^2 \]
\[ = \frac{1}{2} (mr_f^2) \omega_f^2 - \frac{1}{2} (mr_i^2) \omega_i^2 \]
\[ = \frac{1}{2} m (r_f^2 \omega_f^2 - r_i^2 \omega_i^2) \]
\[ = \frac{1}{2} (0.025) \left[ (0.15)^2 (7)^2 - (0.3)^2 (1.75)^2 \right] \text{J} \]
\[ \Delta (kE) = 0.0103 \text{ J} = 1.03 \times 10^{-2} \text{ J} \]

d) \[ W_{\text{total}} = \Delta (kE) \]
\[ W_{\text{total}} = 1.03 \times 10^{-2} \text{ J} \]
### Spinning Figure Skater

\[ \omega_0 = 0.40 \text{ rad/sec} \]

\[ I_0 = 0.40 + \frac{1}{12} (8)(1.8)^2 = 2.56 \text{ kg m}^2 \]

\[ \omega_f = ? \]

\[ I_f = 0.4 + 8(0.25)^2 = 0.90 \text{ kg m}^2 \]

Apply conservation of angular momentum:

\[ I_0 \omega_0 = I_f \omega_f \]

\[ 2.56 \left( \frac{0.4 \text{ rad/sec}}{\text{sec}} \right) = 0.9 \omega_f \]

\[ \omega_f = 1.14 \text{ rad/sec} \]

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### Problems

\[ R_1 = 30.0 \text{ cm} = 0.300 \text{ m} \]

\[ R_2 = 50.0 \text{ cm} = 0.500 \text{ m} \]

\[ I_{cm} = \frac{1}{2} M (R_1^2 + R_2^2) = \frac{1}{2} M (0.3^2 + 0.5^2) = 0.17 M \]

\[ E_i + \sum W_{nc} = E_f \]

\[ Mgh_0 + W_f = \frac{1}{2} MV_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 \]

\[ N(9.8)(2.2) = \frac{1}{2} MV_{cm}^2 + \frac{1}{2} (0.17M) \left( \frac{V_{cm}}{0.5} \right)^2 \]

\[ 21.56 = 0.5 V_{cm}^2 + 0.34 V_{cm}^2 \]

\[ V_{cm} = 5.07 \text{ m/s} \]
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\[
m_A = 4.00 \text{ kg}  \\
m_B = 2.00 \text{ kg}  \\
I_{cm} = 0.300 \text{ kg} \cdot \text{m}^2  \\
R = 0.120 \text{ m} \\
\]

\[
m_A: \quad + \uparrow \Sigma F_y = m_a y  \\
m_A g - T_A = m_A a  \\
4(9.8) - T_A = 4a  \\
39.2 - T_A = 4a \quad \text{eqn #1} \]

\[
m_B: \quad + \uparrow \Sigma F_y = m_b y  \\
T_B - m_B g = m_B a  \\
T_B - 2(9.8) = 2a  \\
T_B - 19.6 = 2a \quad \text{eqn #2} \\
\]

\[
m_c: \quad \uparrow \Sigma \tau_{cm} = I_{cm} \alpha  \\
T_A R - T_B R = I_{cm} \left( \frac{a}{R} \right) \sin \alpha = \frac{a}{R} \\
T_A - T_B = 0.3 a  \\
(0.12)^2  \\
(T_A - T_B) = 20.83 a \quad \text{eqn #3} \\
\]

Solve the 3 equations for the 3 unknowns:

\[
T_A = 36.3 N  \\
T_B = 21.1 N  \\
a = 0.730 \text{ m/s}^2  \\
\]

\[
\alpha = 6.08 \text{ rad/s}^2  \\
\]

The tensions must be different in order to produce a torque that accelerates the pulley when the blocks accelerate.
\[ \text{crate: } m = 50.0 \text{ kg}, \quad a = 1.40 \text{ m/s}^2 \]
\[ \text{cylinder: } R = 0.25 \text{ m}, \quad I = 2.9 \text{ kg m}^2, \quad r = 0.12 \text{ m} \]

\[ F \sin(36.9^\circ) - T \cos(36.9^\circ) = m \alpha \]
\[ n - mg \cos(36.9^\circ) = m(0) \]
\[ n = 5(9.8) \cos(36.9^\circ) \]
\[ n = 39.18 \text{ N} \]

\[ \text{block: } \sum F_y = ma_y \]
\[ n - mg \cos(36.9^\circ) = m(0) \]
\[ n = 5(9.8) \cos(36.9^\circ) \]
\[ n = 39.18 \text{ N} \]

\[ \sum F_x = ma_x \]
\[ m \sin(36.9^\circ) - T = ma \]
\[ T = 5(9.8) \sin(36.9^\circ) - 0.25(3.13) = 5a \]
\[ 19.62 - T = 5a \quad \text{eqn #1} \]
flywheel: \[ \omega^2 \Sigma \zeta = I_{\text{axle}} \times \]

\[ TR = I_{\text{axle}} \frac{a}{R} \]

\[ T(0.2) = 0.5 \frac{a}{0.2} \Rightarrow T = 12.5a \text{ eqn#2} \]

So \( 19.62 - T = 5a \)

\( 19.62 - 12.5a = 5a \)

\( 19.62 = 17.5a \Rightarrow a = 1.124 \text{ m/s}^2 \)

\[ a = 1.12 \frac{m}{s^2} \]

\[ T = 12.5(1.124) \]

\[ T = 14.1 N \]

\[ \text{max} = m \]

\[ \text{radius} = r \]

\[ \text{spherical shell} \]

\[ I = \frac{2}{3} mr^2 \]

\[ g h_0 = \frac{1}{2} V_A^2 + \frac{1}{2} V_B^2 + 2gR \]

\[ g h_0 = \frac{5}{6} V_A^2 + 2gR \]

but also, considering the motion of the spherical shell at point A around the loop:

\[ n = m \frac{V_A^2}{R} \]

\[ mg + n = m \frac{V_A^2}{R} \]

let \( n \to 0 \) for minimum height \( h_0 \Rightarrow \]
\[ mg = m \frac{v_A^2}{R} \]
\[ v_A^2 = gR \]
and plug this into \( g h_0 = \frac{5}{6} v_A^2 + 2gR \)
\[ g h_0 = \frac{5}{6} gR + 2gR \]
\[ h_0 = \frac{17}{6} R \]

b) Point B:
\[ \sum F = ma \]
\[ n' = m \frac{v_B^2}{R} \]
and from conservation of energy eqn:
\[ mg h_0 = \frac{1}{2} m v_B^2 + \frac{1}{2} I w_B^a + mgR \]
\[ mg \left( \frac{17}{6} R \right) = \frac{1}{2} m v_B^2 + \frac{1}{2} (\frac{m}{2} m R) (\frac{v_B}{R})^2 + mgR \]
\[ \frac{17}{6} gR = \frac{5}{6} v_B^2 + gR \]
\[ v_B^2 = \frac{6}{5} \left( \frac{17}{6} gR - gR \right) = \frac{4}{5} gR \]
thus \( n' = \frac{m v_B^2}{R} \) becomes \( n' = \frac{m}{R} \left( \frac{11}{5} gR \right) \)
\[ n' = \frac{11}{5} mg \]
c) Suppose there is no friction on the track. No rotation of spherical shell, only translation so that

\[ E_i = E_f \]

\[ \text{mg} h_0 = \frac{1}{2} \text{mv}^2 + \text{mg} (2R) \]

with \( h_0 = \frac{17}{6} R \)

\[ g \left( \frac{17}{6} R \right) = \frac{1}{2} v_A^2 + 2gR \]

\[ (\frac{17}{6} - 2) gR = \frac{1}{2} v_A^2 \]

\[ \frac{5}{6} gR = \frac{1}{2} v_A^2 \]

\[ v_A = \sqrt{\frac{10}{6} gR} \]

Note that this result is "faster" than when there was friction and the shell would still make the complete loop.

d) Proceeding as in part (c): \( v_A = \sqrt{\frac{10}{6} gR} \) and also

\[ \sum F_i = ma \]

\[ n'' + mg = m \frac{v_A^2}{R} \]

\[ n'' = m \left( \frac{10}{6} gR \right) - mg \]

\[ n'' = \frac{2}{3} mg \quad \text{at point A, no friction.} \]

In part (a) we required \( n = 0 \) at point A when there was friction.
d) As the ball rolls down the rough slope, it gains rotational kinetic energy as well as translational kinetic energy. But as it moves up the smooth slope, its rotational kinetic energy does not change since there is no friction to provide a torque about the CM and change the angular speed.

Let's find first $V_{cm}$ when the ball reaches the bottom:

$$\begin{align*}
E_i &= E_f \\
mgh_0 &= \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}w^2 \\
mgh_0 &= \frac{1}{2}mV_{cm}^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{V_{cm}}{r}\right)^2
\end{align*}$$

$$gH_0 = \frac{3}{2}V_{cm}^2$$

$$V_{cm} = \sqrt{\frac{6}{5}gH_0} \text{ at the bottom level.}$$

Then the ball rolls up the smooth slope to a final height $H$, but its rotational kinetic energy does not change as it rolls upward. Hence,

$$\begin{align*}
E_i &= E_f \\
\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}w^2 &= \frac{1}{2}I_{cm}w^2 + mgH \\
\frac{1}{2}m\left(\frac{6}{5}gH_0\right) &= mgH \\
H &= \frac{3}{5}H_0
\end{align*}$$
b) Some of the initial gravitational potential energy has been converted into rotational kinetic energy so there is less gravitational potential energy at the final height \( H_f \) than at the initial height \( H_0 \). Mechanical energy is conserved throughout the motion. But the initial gravitational potential energy on the rough slope is not all transformed into potential energy on the smooth slope because some of that energy remains as rotational kinetic energy at the highest point on the smooth slope.

\[ \text{Rough section:} \]
\[ MgH_0 = \frac{1}{2} MV_i^2 + \frac{1}{2} I \omega_i^2 + MgH_o \]
\[ MgH_o = \frac{1}{2} MV_i^2 + \frac{1}{2} \left( \frac{2}{5} MR^2 \right) \left( \frac{V_i^2}{R^2} \right) \]
\[ gH_o = (1 + \frac{2}{5})V_i^2 \]
\[ V_i^2 = \frac{5}{7} gH_o \]

\[ \text{Smooth section:} \]
\[ MgH_o + \frac{1}{2} MV_1^2 + \frac{1}{2} I \omega_1^2 = \frac{1}{2} MV_2^2 + \frac{1}{2} I \omega_2^2 \]
\[ \text{But } \omega_1 = \omega_2 \]
\[ MgH_o + \frac{1}{2} MV_1^2 = \frac{1}{2} MGV_2^2 \]
\[ V_2 = \sqrt{\frac{12}{5} gH_o} = \sqrt{\frac{12}{5} (9.8) (50)} \quad \Rightarrow \quad V_2 = 29.0 \text{ m/s} \]

\[ I = \frac{2}{5} MR^2 \]
\[ Vem = WR \]
rolling without slipping
rising to the top of the hill:

\[ E_i = E_f \]

\[ \frac{1}{2} m v_i^2 + \frac{1}{2} I_m \omega_i^2 = m g h + \frac{1}{2} m v_f^2 + \frac{1}{2} I_m \omega_f^2 \]

\[ \frac{1}{2} m v_i^2 + \frac{1}{2} \left( \frac{2}{5} m R^2 \right) \left( \frac{v_i}{R} \right)^2 = m g h + \frac{1}{2} m v_f^2 + \frac{1}{2} \left( \frac{2}{5} m R^2 \right) \left( \frac{v_f}{R} \right)^2 \]

\[ \left( \frac{4}{2} + \frac{1}{5} \right) v_i^2 = g h + \left( \frac{1}{2} + \frac{1}{5} \right) v_f^2 \]

\[ \frac{7}{10} \left( 25 \right)^2 = 9.8 \left( 28 \right) + \frac{7}{10} v_f^2 \]

\[ v_i^2 = 15.26 \text{ m/s} \quad \text{and} \quad v_f = \frac{v_i}{R} = 15.26 \text{ m/s} \]

projectile (free fall) motion:

\[ y_f - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 \]

\[ 0 - 28 = 0 + \frac{1}{2} (-9.8) t^2 \]

\[ t = 3.905 \text{ sec} \]

\[ \Delta x = 15.26 \left( 3.905 \right) \]

\[ \Delta x = 36.5 \text{ m} \]

b) At the bottom of the hill (Point 1), the angular speed of the sphere is \( \omega_i = \frac{v_i}{R} = 25 \)

and this value drops to \( \frac{v_i}{R} = 15.26 \) at the top of the hill. The translational speed \( v_i \) does not change while the ball is in the air (because of zero torque about the center of mass) so just before the ball lands \( v_3 = v_i = 15.26 \) so its total kinetic energy is less rotational and more translational. The translational speed of the ball just before it hits the ground is \( v_3 \) where:

\[ v_{fx} = v_{0x} = 15.26 \text{ m/s} \]

\[ v_{fy} = v_{0y} + a_y t = 0 + 9.8 \left( 3.905 \right) = 32.427 \text{ m/s} \]

\[ v_3 = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(15.26)^2 + (32.427)^2} \]

\[ v_3 = 28 \text{ m/s} \]

which is higher than \( v_i = 25 \text{ m/s} \).
All masses are the same.

System is released from rest.

Note that the angular acceleration of the cylinder is not the same as the angular acceleration of the pulley.

Conservation of total mechanical energy as the block descends a distance of \( y \) gives:

\[
E_i = E_f
\]

\[
mg y = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{pulley} \omega^2 + \frac{1}{2} I_{cylinder} \omega^2 + \frac{1}{2} M v_{cm}^2
\]

block in translation only  
pulley in rotation only  
cylinder in translation and rotation

\[
mg y = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} (Mr^2)(\frac{v_{cm}}{R})^2 + \frac{1}{2} (2Mr^2)(\frac{v_{cm}}{2R})^2 + \frac{1}{2} M v_{cm}^2
\]

Multiply through by 2 to obtain

\[
2gy = 2v_{cm}^2 + v_{cm}^2 + \frac{1}{2} v_{cm}^2 + v_{cm}^2
\]

\[
2gy = 3v_{cm}^2 \quad \Rightarrow \quad v_{cm}^2 = \left( \frac{2g}{3} \right) y
\]

But since the block moves with constant acceleration, and it starts from rest, then the kinematic equation

\[
v_f^2 = v_0^2 + 2a \Delta y
\]

yields \( v_{cm}^2 = 0 + 2ay \) and comparing this equation with yields:

\[
a = \frac{1}{3} g
\]
Apply conservation of total angular momentum:

\[ L_{\text{total}} = L_{\text{total}}^i + L_{\text{rod}} = L_{\text{bird}}^0 + L_{\text{rod}} \]

\[ m\upsilon_0 r^{'} + I_{\text{hinge}}\upsilon_0 = m\upsilon_f r^{'} + I_{\text{hinge}}\upsilon_f \]

\[ m\upsilon_0 r^{'} = \left( \frac{1}{3} ML^2 \right) \upsilon_f \]

\[ 0.5(0.25)(0.5) = \frac{1}{3}(1.5)(0.75)^2 \upsilon_f \]

\[ \upsilon_f = 2.00 \, \text{rad/sec} \]

b) Apply conservation of energy to the motion of the rod after the collision.

\[ \frac{1}{2} I_{\text{hinge}}\upsilon_0^2 + Mg_h L = \frac{1}{2} I_{\text{hinge}}\upsilon_f^2 \]

\[ \frac{1}{2} \left( \frac{1}{3} ML^2 \right) \upsilon_0^2 + Mg \left( \frac{1}{2} L \right) = \frac{1}{2} \left( \frac{1}{3} ML^2 \right) \upsilon_f^2 \]

\[ \frac{1}{6} (0.75)^2 (2)^2 + 9.8(0.25) = \frac{1}{6} (0.75)^2 \upsilon_f^2 \]

\[ \upsilon_f = 6.57 \, \text{rad/sec} \]
The torque of the tension in the string about the hole is 0, so angular momentum is conserved.

\[ L_0 = L_f \]
\[ m_1 v_1 r_1 = m_2 v_2 r_2 \]
\[ 0.8(4) = v_2 r_2 \]
\[ v_2 r_2 = 3.2 \]

Also,
\[ \sum F = ma \]
\[ T = \frac{m v^2}{r} \]

So let \( T = 30 N \), \( v = v_2 \) and \( r = r_2 \) to obtain
\[ 30 = 0.25 \frac{1}{r_2^2} \]
and use \( v_2 = 3.2 \frac{r_2}{r_2} \)
\[ 30 = 0.25 \left( \frac{3.2}{r_2} \right)^2 \]
\[ r_2^3 = 0.0853 \]

\[ r_2 = 0.440 \text{ m} = 44.0 \text{ cm} \]
Let's choose the counterclockwise direction to be the positive rotational sense. Apply conservation of total angular momentum:

\[ L_{\text{before}} = L_{\text{after}} \]

\[ I_{\text{runner}} + I_{\text{table}} = I_{\text{runner}} + I_{\text{table}} \]

\[ m_v R - I_{\text{table}} w_0 = m v_f R + I_{\text{table}} w_f \]

\[ 55(2.8)(3) - 80(0.2) = 55(3w_f)(3) + 80 w_f \]

\[ 446 = 575 w_f \]

\[ w_f = 0.776 \text{ rad sec}^{-1} \]

Note that the answer came out positive. This means that the final angular velocity is in the direction that we assumed, namely, counterclockwise.
Center of Percussion

\[ M = 0.8 \text{ kg} \]
\[ I_{cm} = 0.0543 \text{ kg m}^2 \]

Let \( F \) be the force exerted by the ball on the bat at a distance \( x \) from the end of the bat.

If the impact of the ball on the bat is such that the end of the bat is to remain at rest as the bat begins to move, then there will be pure rotation about the end of the bat so that the linear acceleration \( a_{cm} \) of the center of mass of the bat is

\[ a_{cm} = \alpha x_{cm} \]

Thus

\[ \sum F_{ext} = M a_{cm} \]
\[ F = M a_{cm} \]

And

\[ I_{end} \alpha = F x = I_{end} \alpha \]
\[ \text{but } F = M a_{cm} \]
\[ \text{and } I_{end} = I_{cm} + M x_{cm}^2 \]

from the parallel axis theorem.

\[ M a_{cm} x = (I_{cm} + M x_{cm}^2) \alpha \]
\[ M x_{cm} x = (I_{cm} + M x_{cm}^2) \alpha \]

Cancel out the \( \alpha \) so that

\[ x = \frac{I_{cm} + M x_{cm}^2}{M x_{cm}} \]

\[ x = x_{cm} + \frac{I_{cm}}{M x_{cm}} \]
\[ x_{cm} = 0.600 \, \text{m} \]

\[ I_{cm} = 0.053 \, \text{kg} \cdot \text{m}^2 \]

\[ M = 0.8 \, \text{kg} \]

\[ x = 0.6 + \frac{0.053}{0.8 \cdot 0.6} \]

\[ x = 0.710 \, \text{meters} \]

Note that the center of percussion (also called the "sweet spot") is farther from the handle than the center of mass.