Exercises: 4, 9, 11, 17, 19, 30, 34, 47, 51
Problems: 72, 95, 97, 99, 101

Exercises

4 See figure E14.4 on page 464 of the textbook.

a) \( T = \frac{2}{8\text{ sec}} = 0.25\text{ sec} = \text{ Period} \)
\[ f = \frac{1}{T} = \frac{1}{0.25} \text{ Hz} \]
\[ f = 4 \text{ Hz} \]

b) \( A = 10.0 \text{ cm} \)

c) \( T = 16.0 \text{ sec} \)

d) \( w = 2\pi f \)
\[ w = 2\pi \left( 0.0625 \right) \text{ rad/sec} \]
\[ w = 0.393 \text{ rad/sec} \]

9 \( T = 0.900 \text{ sec} \Rightarrow w = 2\pi f = \frac{2\pi}{T} = 6.981 \text{ rad/sec} \)
\( A = 0.320 \text{ m} \)
\( x = 0.320 \text{ m at } t = 0 \text{ sec} \). Thus \( x(t) = A \cos(wt) \)

(a) time to go from \( x = 0.320 \text{ m} \) to \( x = 0.160 \text{ m} \),
\[ x(t) = 0.320 \cos \left( 6.981 t \right) \]
\[ 0.16 = 0.32 \cos \left( 6.981 t \right) \]
\[ 0.5 = \cos \left( 6.981 t \right) \]
\[ 6.981 t = \cos^{-1} \left( 0.5 \right) = 60^\circ = \frac{\pi}{3} \text{ radians} \]
\[ 6.981 t = \frac{\pi}{3} \text{ rad} \]
\[ t = \frac{\pi}{3(6.981)} \]
\[ t = 0.150 \text{ sec} \]
(b) Time to go from $x = 0.160 \text{ m}$ to $x = 0$. If $x = 0$ is $\frac{1}{4}$ of the period, then $T = \frac{0.9 \text{ sec}}{4} = 0.225 \text{ sec}$. We found that the time to go from $x = A = 0.320 \text{ m}$ to $x = 0.160 \text{ m} = \frac{A}{2}$ is $0.150 \text{ sec}$. Therefore, the time to go from $x = 0.160 \text{ m}$ to $x = 0$ is $0.225 \text{ sec} - 0.150 \text{ sec} = 0.075 \text{ sec}$.

\[ \Delta t = (0.225 - 0.150) \text{ sec} = 0.075 \text{ sec} \]

11. \( m = 2 \text{ kg}, \quad k = 300 \text{ N/m} \)

\[ v_{\text{max}} = 12 \text{ m/s} \quad \text{in negative direction (compressing the spring)} \]

at \( t = 0 \). Thus \( x = -A \sin(wt) \)

a) \[ \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} k A^2 \]

\[ A = \sqrt{\frac{m}{k}} v_{\text{max}} = \sqrt{\frac{2}{300}} \quad (12) \Rightarrow A = 0.980 \text{ m} \]

b) \[ x(t) = -A \sin(wt) \quad \text{which also equals} \quad x(t) = A \cos(wt + \phi) \]

So set \(-A \sin(wt) = A \cos(wt + \phi)\),

\[ -\sin(wt) = \cos(wt) \cos(\phi) - \sin(wt) \sin(\phi) \]

This equation is satisfied if \( \phi = 90^\circ \equiv \frac{\pi}{2} \).

Alternatively, \[ \frac{x}{x_0} \rightarrow t \]
c) \( x(t) = -A \sin(\omega t) \)
\[
\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{2}}
\]
\( x(t) = (-0.980 \text{ m}) \sin(12.2t) \)  \( \omega = 12.2 \text{ rad/sec} \)

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\( k = 2.50 \frac{N}{\text{cm}} = 250 \frac{N}{\text{m}} \)

See figure E 14.17 on page 464 of the textbook.

a) Note that \( T = 0.20 \text{ sec} \)
\[
T = \frac{2\pi}{\sqrt{\frac{m}{k}}}
\]
\[0.20 = 2\pi \sqrt{\frac{m}{250}} \implies m = 0.253 \text{ kg}\]

b) \( A = -\omega^2 x \)
\[|A_{\text{max}}| = +\frac{k}{m} A \quad \text{note that} \quad A_{\text{max}} = 12 \text{ m/s}^2 \]
\[12 = 250 \frac{A}{0.253} \quad \implies \quad A = 0.0122 \text{ m} = 1.22 \text{ cm} \]

c) \( F_{\text{max}}^{\text{spring}} = kA \)
\[F_{\text{max}}^{\text{spring}} = 250 \left(0.0122\right) N \]
\[F_{\text{max}}^{\text{spring}} = 3.04 N \]

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\( m = 1.50 \text{ kg} \)
\( x(t) = (7.40 \text{ cm}) \cos \left[ 4.16 t - 2.42 \right] \)

a) \( \omega = 4.16 \text{ rad/sec} \)
\[
T = \frac{2\pi}{\omega} = \frac{2\pi}{4.16} \implies T = 1.51 \text{ sec} \]

b) \( \omega = \sqrt{\frac{k}{m}} \implies 4.16 = \sqrt{\frac{k}{1.50}} \implies k = 26.0 \frac{N}{\text{m}} \)

c) \[v = \frac{dx}{dt} = -\left(0.074 \frac{\text{meters}}{\text{sec}}\right) \sin \left[ 4.16 t - 2.42 \right] (4.16) \]
\[\text{note that} \quad v_{\text{max}} = (0.074)(4.16) \frac{\text{m}}{\text{sec}} \implies v_{\text{max}} = 0.308 \text{ m/sec} = 30.8 \text{ cm/sec} \]
d) \[ A = 7.40 \text{ cm} = 0.0740 \text{ m} \]
\[ F_{\text{max}} = kA = 26(0.074) \text{ N} \]
\[ F_{\text{max}} = 1.92 \text{ N} \]

\[ t = 1 \text{ sec} \]
\[ x = 0.074 \cos \left[ 4.16(1) - 2.42 \right] \Rightarrow x = -0.0125 \text{ m} = 1.25 \text{ cm} \]
\[ v = \frac{dx}{dt} = (-0.074)(4.16) \sin \left( 4.16(1) - 2.42 \right) \Rightarrow v = -0.303 \text{ m/ sec} \]

\[ a = \frac{dv}{dt} = -\omega^2 x = -\left( 4.16 \right)(-0.0125) \]
\[ a = 0.216 \text{ m/ sec}^2 \text{ at } t = 1 \text{ sec} \]

\[ F = m a \]
\[ F = 1.5(0.216) \text{ N} \]
\[ F = 0.324 \text{ N} \text{ at } t = 1 \text{ sec} \]

\[ m = 0.15 \text{ kg} \]
\[ k = 300 \text{ N/ m} \]
\[ \text{at } x = 0.012 \text{ m the speed is } \dot{v} = 0.300 \text{ m/s} \]

(a) \[ E = \frac{1}{2} kx^2 + \frac{1}{2} \frac{1}{m} \dot{v}^2 \]
\[ E = \frac{1}{2} \left( 300 \right)(0.012)^2 + \frac{1}{2} \left(0.15\right)(0.3)^2 \]
\[ E = 0.0284 \text{ J} \]

(b) \[ E = \frac{1}{2} kA^2 \]
\[ 0.0284 = \frac{1}{2} \left(300\right)A^2 \Rightarrow A = 0.014 \text{ m} \]

(c) \[ E = \frac{1}{2} m \dot{v}_{\text{max}}^2 \]
\[ 0.0284 = \frac{1}{2} \left(0.15\right) \dot{v}_{\text{max}}^2 \Rightarrow \dot{v}_{\text{max}} = 0.615 \text{ m/ sec} \]
34. \( k = 75 \text{ N/m} \)  

See figure E14.34 in the textbook on pp. 465

\( a) \quad T = 1.6 \text{ sec} \)

\( b) \quad f = \frac{1}{T} = \frac{1}{1.6} \Rightarrow f = 0.625 \text{ Hz} \)

\( c) \quad \omega = 2\pi f = 2\pi (0.625) \Rightarrow \omega = 3.93 \text{ rad/sec} \)

\( d) \quad v_{\text{max}} = 20 \text{ cm/sec} = 0.20 \text{ m/sec} \)

\[ \frac{1}{2} kA^2 = \frac{1}{2} m v_{\text{max}}^2 \quad \Rightarrow \quad A = \sqrt{\frac{m}{k}} \quad v_{\text{max}} = \frac{v_{\text{max}}}{\omega} = \frac{0.2}{3.93} \]

\[ A = 0.051 \text{ m} \]

The amplitude is reached at those times when \( v = 0 \). That is, at \( t = 0.4 \text{ sec}, 1.2 \text{ sec}, 2.0 \text{ sec}, \ldots \)

\( e) \quad a_{\text{max}} = \omega^2 A \)

\[ a_{\text{max}} = (3.93)^2 (0.051) \]

\[ a_{\text{max}} = 0.79 \text{ m/sec}^2 = 79 \text{ cm/sec}^2 \]

The maximum acceleration occurs when \( x = \pm A \), that is, at \( t = 0.4 \text{ sec}, 1.2 \text{ sec}, 2.0 \text{ sec}, \ldots \) Note that there is a mistake in the numbers on the graph.

\( f) \quad T = 2\pi \sqrt{\frac{m}{k}} \)

\[ 1.6 = 2\pi \sqrt{\frac{m}{75}} \quad \Rightarrow \quad m = \frac{4.9}{1.5} \text{ kg} \]

47. \( m = 2.35 \text{ kg} \)

\( L = 1.050 \text{ m} \)

Model the system as a simple pendulum oscillating with small amplitude. The number of swings per second is the frequency \( f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{L}} = \frac{1}{2\pi} \sqrt{\frac{9.8}{1.05}} \)

\[ f = 0.407 \text{ swings per second} \]
Simple Pendulum

L = 2 m
θ_{max} = 30°

a) For small amplitude oscillations,

\[ T = 2\pi \sqrt{\frac{L}{g}} \]

\[ T = 2\pi \sqrt{\frac{2}{9.8}} \]

\[ \Rightarrow T = 2.84 \text{ sec} \]

b) Large amplitude oscillations,

\[ T = 2\pi \sqrt{\frac{L}{g}} \left\{ 1 + \frac{1}{4} \sin^2 \left( \frac{30°}{2} \right) + \frac{9}{64} \sin^4 \left( \frac{30°}{2} \right) + \frac{225}{2304} \sin^6 \left( \frac{30°}{2} \right) + \cdots \right\} \]

\[ T = 2.84 \left\{ 1 + \frac{1}{4} \sin^2 (15°) + \frac{9}{64} \sin^4 (15°) + \frac{225}{2304} \sin^6 (15°) \right\} \]

\[ T = 2.89 \text{ sec} \]

The formula used in part (b) is more accurate than the formula used in part (a).

The formula in part (a) is in error by

\[ \% \text{ error} = \frac{(2.84 - 2.89)}{2.89} \times 100 \]

\[ \% \text{ error} = -2\% \]
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\[ T = 2.0 \text{ sec} \] for a physical pendulum.

Use Parallel-axis theorem:

\[ I_p = I_{cm} + Md^2 \]
\[ I_p = MR^2 + MR^2 \]
\[ I_p = 2MR^2 \]

\[ d = R \]

\[ T = 2\pi \sqrt{\frac{I_p}{MgR}} \]  
\[ a = 2\pi \sqrt{\frac{2MR^2}{MgR}} \]
\[ l = \pi \sqrt{\frac{2R}{g}} \]
\[ \frac{1}{\pi^2} = 2R \]
\[ R = \frac{g}{2a\pi^2} \]

\[ R = 9.8 \]
\[ R = 0.496 \text{ meters} \]

Problems

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Free-body diagram of second mass \( m \):

\[ a_\text{max} = \omega^2 A \]
\[ A_\text{max} = \frac{kA}{(M+m)} \]

\[ f_s^\text{max} = mA_\text{max} \]

\[ \Sigma F_y = m \rightarrow 0 \]
\[ n = mg \rightarrow 0 \]
\[ f_s^\text{max} = m \cdot k 
\]

\[ \Sigma F_x = mA_\text{max} \]

\[ n = mg \]

\[ M_n = m \cdot k \cdot A \]

\[ A = M_s g \frac{(M+m)}{k} \]
First: Calculate the speed with which the ball \( m = 2 \, \text{kg} \) hits the ball with \( m' = 3 \, \text{kg} \).

Use conservation of mechanical energy:

\[
\frac{1}{2} m v_i^2 + mgh_0 = \frac{1}{2} m v_f^2 + mgh_f
\]

\[
\frac{1}{2} v_i^2 = gh_0 \Rightarrow v_i = \sqrt{2gh_0}
\]

\[
v_f = \sqrt{v_i^2 - 2gh_0} = 1.4 \, \text{m/s}
\]

Second: Consider the collision (completely inelastic) and use conservation of total linear momentum to find the speed of both balls (together) right after the collision:

\[
m' = 3 \, \text{kg} \quad \quad v = 0
\]

\[
m = 2 \, \text{kg} \quad \quad v = 1.4 \, \text{m/s}
\]

\[
M = 5 \, \text{kg}
\]

Before collision:

\[
3v(0) + 2(1.4) = 5v
\]

\[
v = 0.56 \, \text{m/s}
\]

After collision:

Third: Calculate maximum height reach by both balls together by using conservation of mechanical energy:

\[
\frac{1}{2} Mv^2 = Mgh_f
\]

\[
\frac{1}{2}(0.56)^2 = 9.8 \, h_f \Rightarrow h_f = 0.016 \, \text{m}
\]

\[
\cos \theta_{\text{max}} = \frac{L - h_f}{L} = 0.5 - 0.016 \Rightarrow 0.484
\]

\[
\theta_{\text{max}} = 14.5^\circ
\]
The frequency of oscillation is given by

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} = \frac{1}{2\pi} \sqrt{\frac{9.8}{0.5}} = 0.705 \text{ Hz} = f \]

since the angular amplitude is relatively small.

Uniform rod of mass \( M \) and length \( L \). Consider a free-body diagram of the rod and apply Newton’s Second Law of motion for rotation:

\[ \sum \tau_{cm} = I_{cm} \alpha \]

where

\[ I_{cm} = \frac{1}{12} ML^2 \]

\[ \alpha = \frac{d^2 \theta}{dt^2} \]

Thus

\[ \sum \tau_{cm} = \frac{1}{12} ML^2 \frac{d^2 \theta}{dt^2} \]

\[ -F_{spring} \frac{L}{2} \cos(\theta) = \frac{ML^2}{12} \frac{d^2 \theta}{dt^2} \]

\[ L \rightarrow \text{lever arm} \]

\[ -k \frac{L}{2} \cos(\theta) = \frac{ML^2}{12} \frac{d^2 \theta}{dt^2} \]

\[ \frac{L}{2} \sin(\theta) = x \] from geometry

\[ -k \frac{L}{2} \sin(\theta) \frac{L}{2} \cos(\theta) = \frac{ML^2}{12} \frac{d^2 \theta}{dt^2} \]

\[ \frac{d^2 \theta}{dt^2} + \frac{3K}{m} \sin(\theta) \cos(\theta) = 0 \]

For small angular oscillations, \( \sin \theta \approx \theta \), hence \( \cos \theta \approx 1 \).
the equation of motion becomes
\[ \frac{d^2 \theta}{dt^2} + \frac{3k}{M} \theta = 0 \]
\[ \text{Motion is } \sim \text{ Simple Harmonic} \]
comparing with
\[ \frac{d^2 \theta}{dt^2} + \omega^2 \theta = 0 \]
yields
\[ \omega^2 = \frac{3k}{M} \]
but since \( T = \frac{2\pi}{\omega} \)
finally
\[ T = 2\pi \sqrt{\frac{M}{3k}} \]

First let's calculate the center of mass of the L-shaped object.

\[ x_{\text{cm}} = \frac{\sum x_i m_i}{\sum m_i} = \frac{(\frac{L}{2}) M + (0) M}{M + M} = \frac{L}{2} \frac{M}{2M} = \frac{L}{4} \]
\[ y_{\text{cm}} = \frac{\sum y_i m_i}{\sum m_i} = \frac{(0) M + (\frac{L}{2}) M}{M + M} = \frac{\frac{L}{2} M}{2M} = \frac{L}{4} \]
Thus
\[ d = \sqrt{(\frac{L}{4})^2 + (\frac{L}{4})^2} = \frac{L}{2\sqrt{2}} \]
The frequency of oscillation of the physical pendulum is given by

\[ f = \frac{1}{2\pi} \sqrt{\frac{Mg d}{I_P}} \]

where

\[ I_P = \text{moment of inertia of physical pendulum about the pivot} \]
\[ d = \text{distance from the center of mass to the pivot} \]

we already found that \( d = \frac{L}{2\sqrt{2}} \).

For a rod of length \( L \) and mass \( M \), the moment of inertia about an axis through one end and \( L \) to the rod is \( \frac{1}{3} ML^2 \), but here we have 2 rods, thus

\[ I_P = \frac{1}{3} ML^2 + \frac{1}{3} ML^2 \]

\[ I_P = \frac{2}{3} ML^2 \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{(2M)g - \left(\frac{1}{2}\sqrt{2}\right)}{\frac{2}{3} ML^2}} \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{3}{2\sqrt{2}} \frac{g}{L}} \]

\[ f = \frac{1}{4\pi} \sqrt{\frac{6}{\sqrt{2}} \frac{g}{L}} \]
The Effective Spring Constant of Two Springs

(a) \[ \Sigma F_x = -k_1 x - k_2 x \]
\[ \Sigma F_x = -(k_1 + k_2) x \]

\[ \Sigma F_x = -k_{\text{eff}} x \]

\[ k_{\text{eff}} = k_1 + k_2 \]

(b) \[ \Sigma F_x = -k_1 x - k_2 x \]
\[ \Sigma F_x = -(k_1 + k_2) x \]

\[ k_{\text{eff}} = k_1 + k_2 \]

(c) For the block \( m \):
\[ \Sigma F_x = -k_2 (x - x_1) \]
\[ \Sigma F_x = -k_2 x + k_2 \left( \frac{k_1 x}{k_1 + k_2} \right) \]
\[ \Sigma F_x = -k_2 x + k_2 \left( \frac{k_1 x}{k_1 + k_2} \right) \]

For the point \( p \):
\[ k_1 x_1 - k_2 (x - x_1) = 0 \]
\[ k_1 x_1 - k_2 x + k_2 x_1 = 0 \]

\[ x_1 = \frac{k_2 x}{k_1 + k_2} \]

So
\[ k_{\text{eff}} = k_2 - \frac{k_2^2}{k_1 + k_2} = k_2 \left( \frac{k_1}{k_1 + k_2} \right) - \frac{k_2}{k_1 + k_2} \]
\[ k_{\text{eff}} = \frac{k_1 k_2}{k_1 + k_2} \]
(d) if we have a spring with spring constant $k$,

$$\frac{k}{2} = \frac{k'\cdot k''}{k' + k''}$$

and we cut it in half, then we have a spring with spring constant $k_1 = k_2 = k'$ where

$$k_{\text{eff}} = \frac{k_1 \cdot k_2}{k_1 + k_2}$$

$$k'' = \frac{(k')^2}{k' + k'}$$

$$k = \frac{(k')^2}{2k'}$$

$$k = \frac{k'}{2} \quad \therefore \quad k' = 2k$$

So the spring constant of half a spring equals twice the spring constant of the original whole spring. The frequency of oscillation is in general

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

so if the spring constant doubles, the frequency of oscillation increases by a factor of $\sqrt{2}$. 
